Engineering Mechanics / Statik

Newton's laws

i) A particle remains at rest or continues to move with uniform velocity (in straight line with constant speed) if there is no unbalanced froce acting on it.

ii) The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.

iii) The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction.

$$\boxed{F = ma}$$

F: is the vector sum forces acting on the partical

a: is the resulting acceleration.

m: mass of partical.

Units

Quantity	Dimensional Symbol	SI unit: unit	SI unit: Symbol	U.S unit: unit	U.S unit: Symbol
Mass	M	Kilogram	kg	Slug	—
Length	L	meter	m	foot	f
Time	T	Second	S	Second	S
Force	F	newton	N	pound	I

$$N = kg \cdot m / S^2$$

$$W(N) = m(kg) \times g(m/s^2)$$

weight

$KN = 1000\ N$, $MN = 1\,000\,000\ N = 10^6 N$

$GN = 10^9 N$

$m = 1000\ mm = 100 cm$

$cm = 10\ mm$

$kg = 10^3 gr$

$min = 60\ S$, $hour = 60\ min$

Law of Gravitation

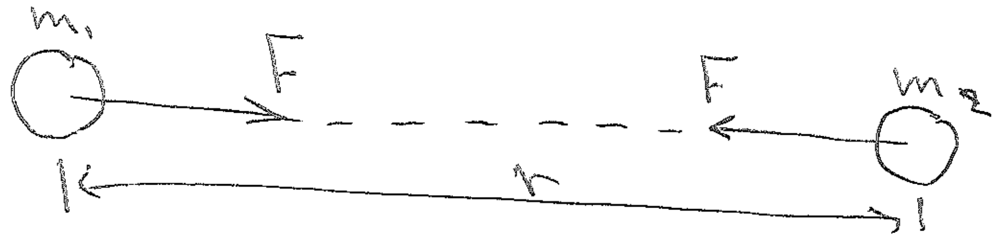
$$F = G \frac{m_1 m_2}{r^2}$$

F: the mutual force of attention between two particles.

G: a universal constant know as the constant of gravitation. (6.673×10^{-11} $m^3/Kg \cdot s^2$)

m_1 & m_2: the masses of the two perticals

r: the distance between the centers of the particles.



$$W = mg$$

W: weight (N)

m: mass (Kg)

g: acceleration of gravity m/s^2

Ex

Determine the weight in newtons of a car whose mass is 1400 kg.

$W = mg = 1400 \times 9.81 = 13.730\ N$

Ex

Use Newton's law of universal gravitation to calculate the weight of a 70 kg person standing on the surface of the earth. Then repeat the calculation by using $W = mg$ and compare your two results.

$r = 6371\ Km$

$m_1 = 5.976 \times 10^{24}$, $m_2 = 70\ kg$

$$W = \frac{G m_1 m_2}{r^2} = \frac{(6.673 \times 10^{-11})(5.976 \times 10^{24}) \times 70}{(6371 \times 10^3)^2}$$

$\boxed{W = 688 N}$

$W = mg = 70(9.81) = \boxed{687 N}$

Basic Concepts

Space: is the geometric region Occupied by bodies whose positions are described by linear and angular measurements relative to a coordinates system. For three-dimensional problems, three independent Coordinates are needed. For two-dimensional problems, only two coordinates are required.

Time is the measure of the succession تعاقب of events and is a basic quantity in dynamics. Time is not directly involved in analysis of static problems.

Mass is a measure of the inertia of a body which is its resistance to a change of velocity. Mass can also be throught of as a the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in static.

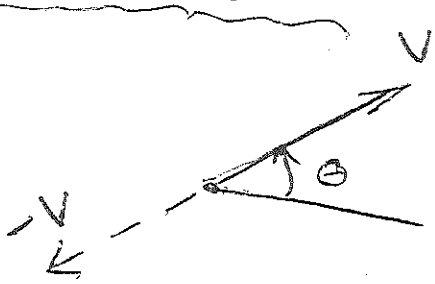
Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of the force is characterized by its magnitude and its by the direction of its action

Particle is a body of negligible dimensions. in the mathematical Senee, a partical is a body whose dimensions are considered to be near zero so that ~~that~~ we may analyze it as a mass concentrated at a point

Rigid body A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand.

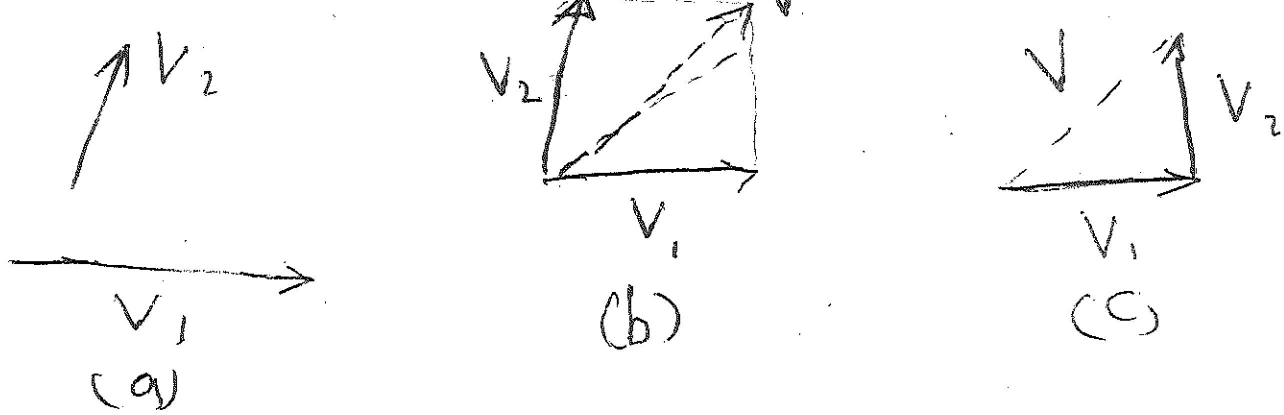
Conventions for Equations and Digrams

A vector quantity V is represented by line as shown in the figure. it has direction and has an arrow head to indicate the direction.



Length of Vector directed line

length: $|V|$



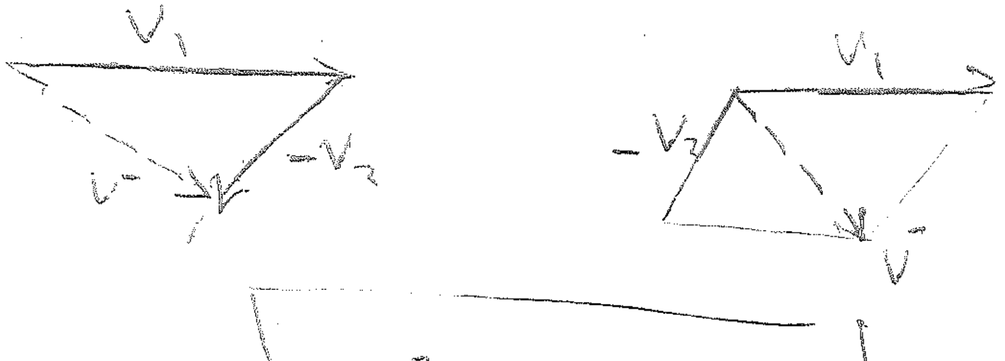
From figure (a) said free vectors may be replaced by thir equivalent vector V which is the diagonal of the parallelogram formed by vectors V_1 and V_2 as its two sides as shown in (b) and called the vector sum.

$$\boxed{V = V_1 + V_2}$$

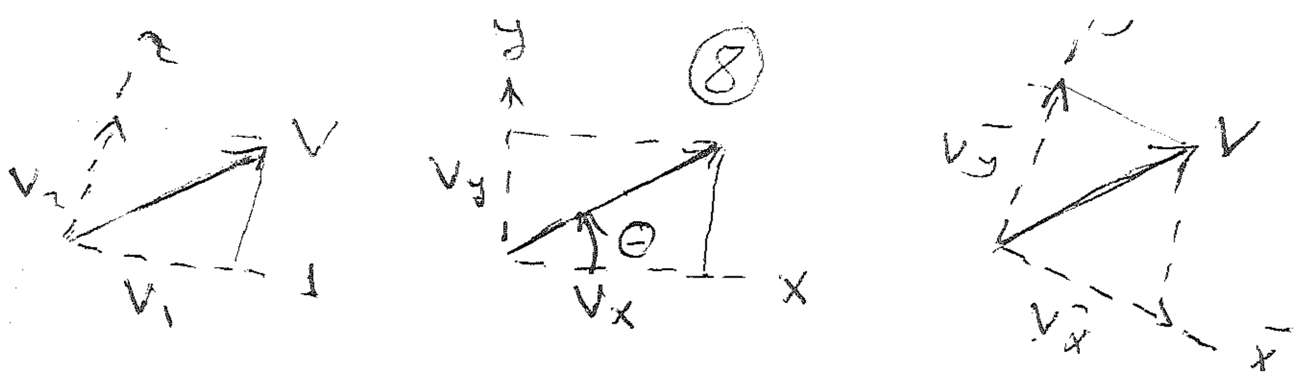
In figure (c) we can obtain V as

$$\boxed{V = V_1 + V_2 = V_2 + V_1}$$

The difference $V_1 - V_2$ between the two vectors is easily obtained by adding V_2 to V_1 as shown in fig below



$$\boxed{V' = V_1 - V_2}$$



$$\boxed{\theta = \tan^{-1}\frac{V_y}{V_x}}$$

$$\boxed{\mathbf{V} = V\mathbf{n}}$$

$\mathbf{V}$: vector V: magnitude $\mathbf{n}$: a vector

$\mathbf{n}$: called unit vector

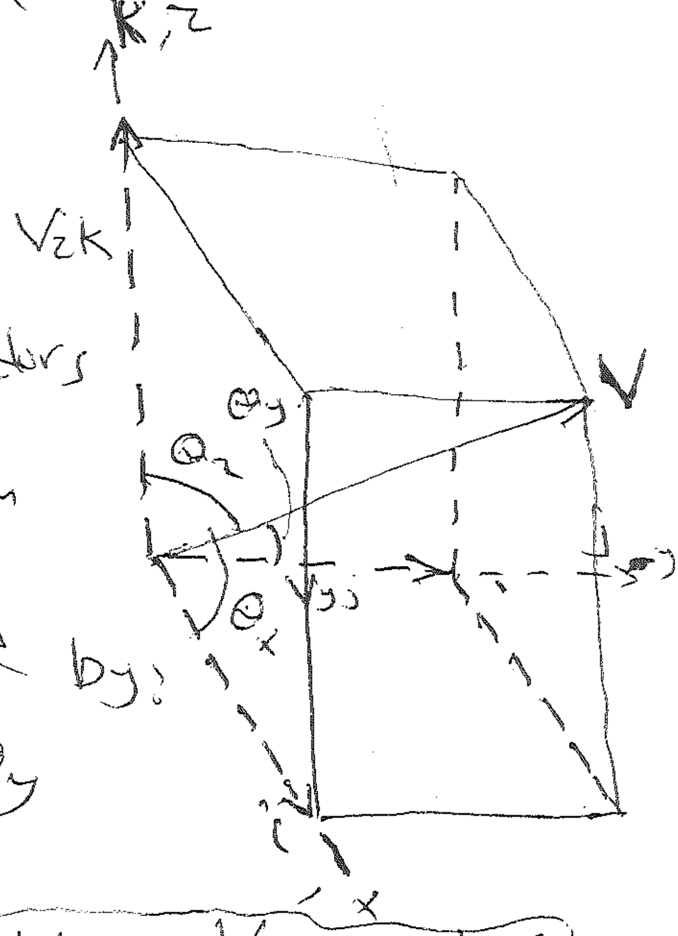
$$\boxed{\mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j} + V_z \mathbf{k}}$$

$\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors

We now make use direction cosines l, m, n of $\mathbf{V}$ which are defined by:

$l = \cos\theta_x$, $m = \cos\theta_y$

$n = \cos\theta_z$



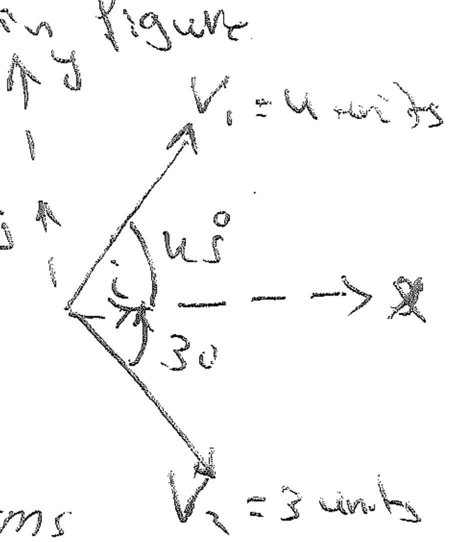
so $V_x = lV$, $V_y = mV$, $V_z = nV$

$V^2 = V_x^2 + V_y^2 + V_z^2$ ← from Pythagorean theorem

$l^2 + m^2 + n^2 = 1$

Example

For the vectors V_1 and V_2 shown in figure



(a) determine the magnitude S of their vector sum $S = V_1 + V_2$

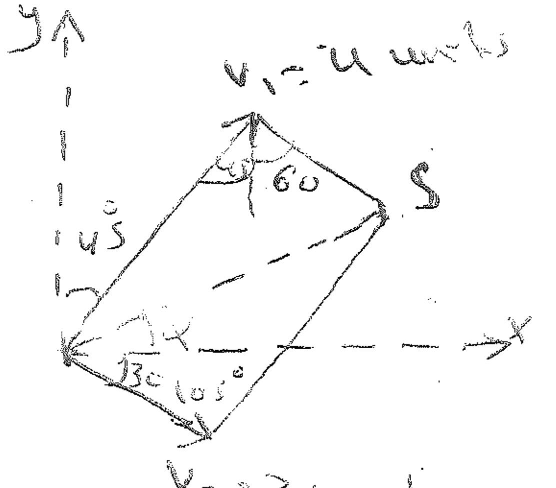
(b) determine the angle α between S and the positive x-axis

(c) write S as a vector in terms the unit vectors i and j and then write a unit vector n along the vector sum S

(d) Determine the ~~distance~~ vector difference

$$D = V_2 - V_1$$

Sol



(a) We construct to scale the parallelogram shown in fig for adding V_1 and V_2 using the law of cosines.

$$\boxed{S^2 = V_2^2 + V_1^2 - 2V_2V_1\cos\theta}$$

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105°$$

$$\boxed{S = 5.59 \text{ units}}$$

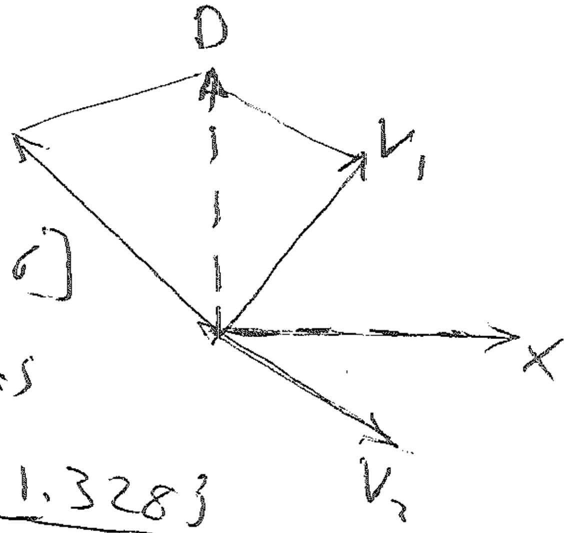
(b) Using the law of sines for the lower triangle:

$$\frac{\sin 105}{5.59} = \frac{\sin(\alpha + 30)}{4}$$

$$\sin(\alpha + 30) = 0.697$$

$$(\alpha + 30) = 43.8^\circ \quad \therefore \boxed{\alpha = 13.76^\circ}$$

(c)



$$\mathbf{S} = \overset{5.59}{S}[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$

$$= S[\mathbf{i}\cos 13.76 + \mathbf{j}\sin 13.76]$$

$$= 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units}$$

Then $\mathbf{n} = \dfrac{\mathbf{S}}{S} = \dfrac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59}$

$\therefore$ $\mathbf{n} = 0.971\mathbf{i} + 0.238\mathbf{j}$

(d) The vector difference $\mathbf{D}$ is

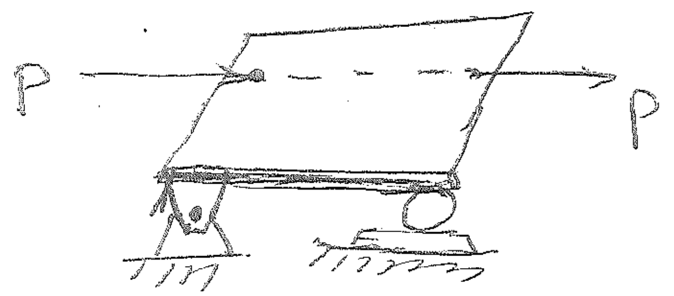
$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 \longleftarrow \mathbf{V}_1 + (-\mathbf{V}_2)$$

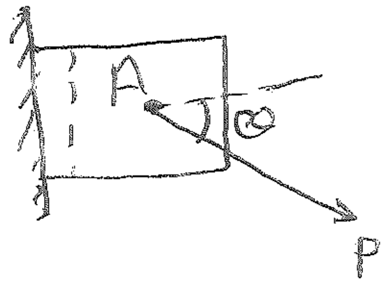
$$= 4(\mathbf{i}\cos 45 + \mathbf{j}\sin 45) - 3(\mathbf{i}\cos 30 - \mathbf{j}\sin 30)$$

$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units}$$

Force System (II)

Force

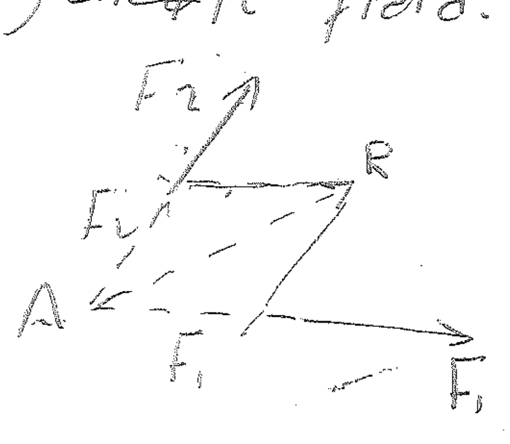


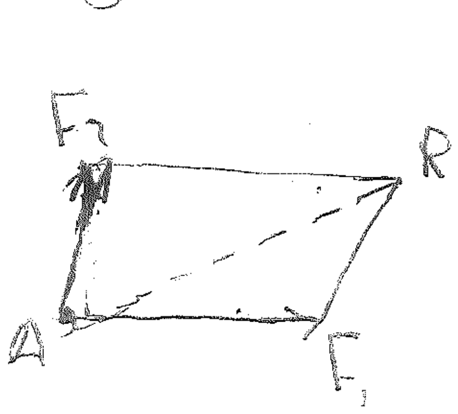


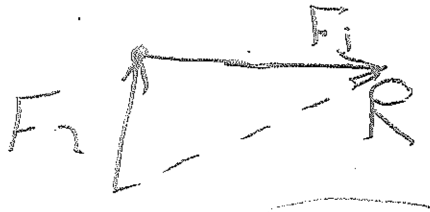
Force classification

(1) contact force / produced by direct physical contact.

(2) body force / generated by virtue (استناد) of the position of a body within a force field such as a gravitational, electric or magenetic field. our weight







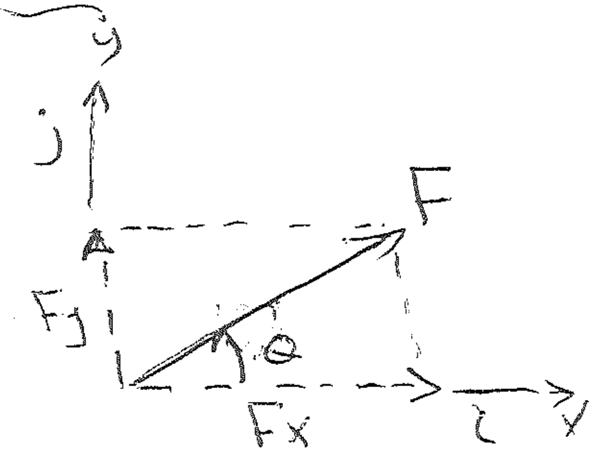
vector equation ← $\boxed{R = F_1 + F_2}$

Rectangular Components

$$\boxed{F = F_x + F_y}$$

where F_x and F_y are vector components

$$\boxed{F = F_x i + F_y j}$$



F_x and F_y the x and y scalar components of the Vector F.

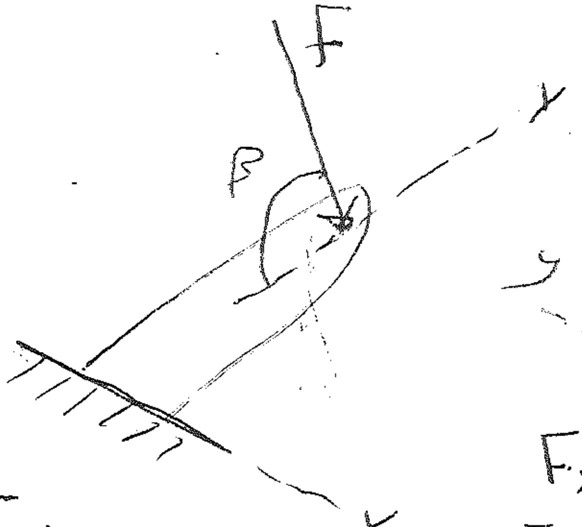
From the figure:

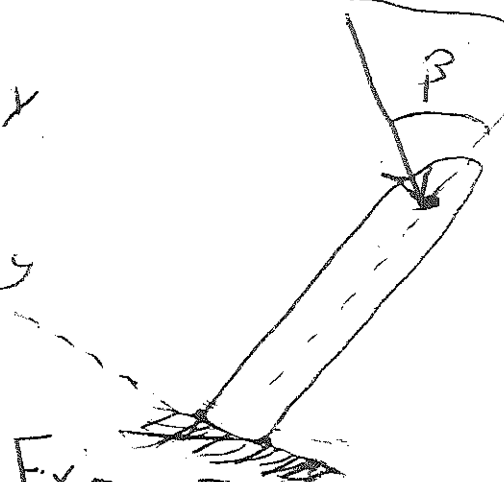
$$\boxed{\begin{aligned} F_x &= F\cos\theta \\ F_y &= F\sin\theta \end{aligned}}$$

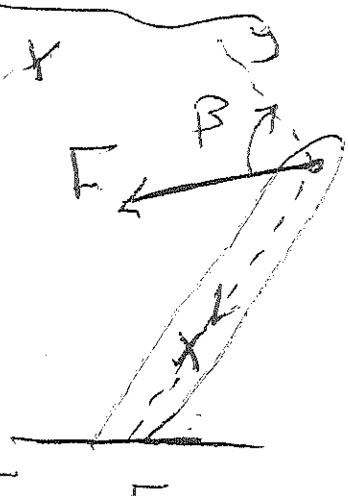
or

$$\boxed{\begin{aligned} F &= \sqrt{F_x^2 + F_y^2} \\ \theta &= \tan^{-1}\frac{F_y}{F_x} \end{aligned}}$$

Determine the components of a force







$F_x = F\sin(\pi-\beta)$

$F_y = -F\cos(\pi-\beta)$

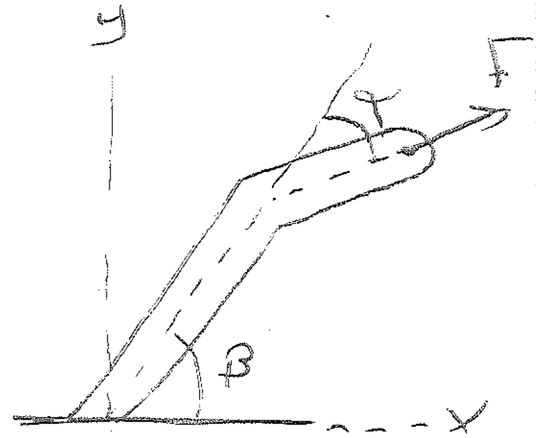
$F_x = -F\cos\beta$

$F_y = -F\sin\beta$

$F_x = F\sin\beta$

$F_y = F\cos\beta$

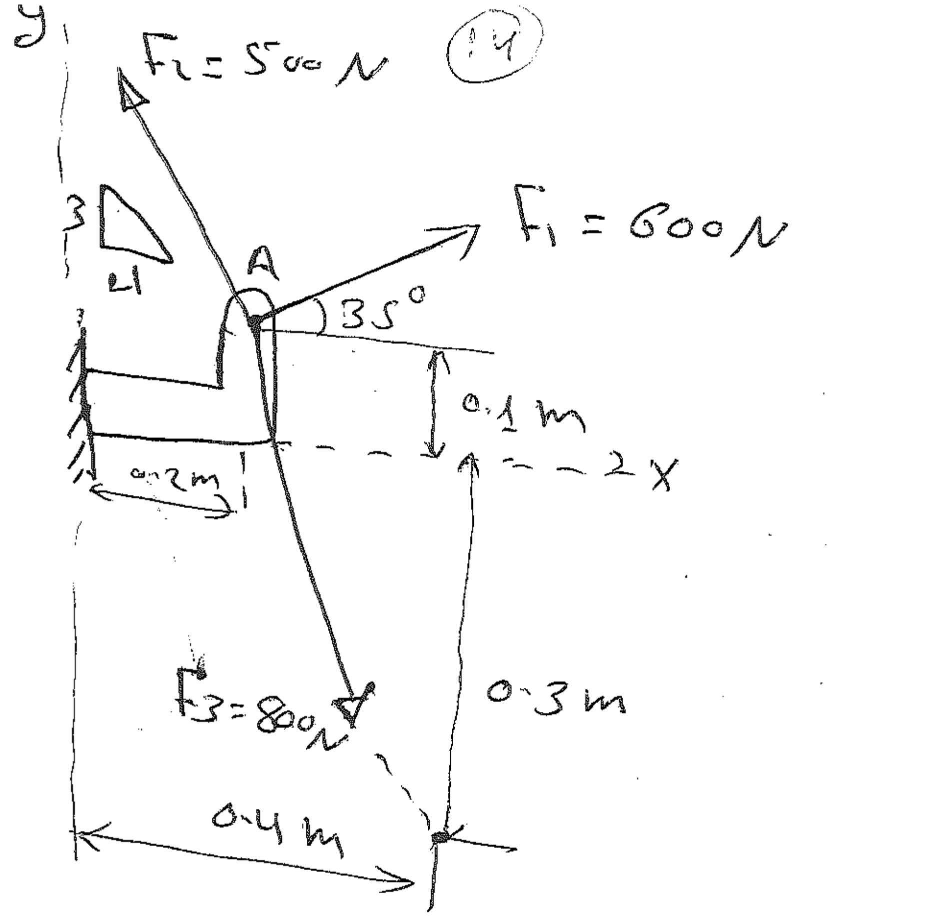
$$F_x = F\cos(\beta - \alpha)$$
$$F_y = F\sin(\beta - \alpha)$$



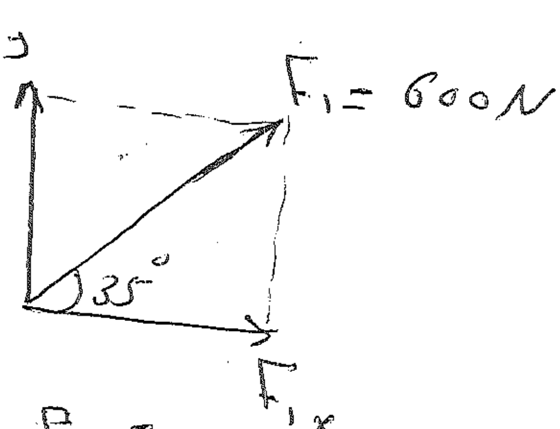
ΣF_x means the algebre sum of the x scalar components.

$$\Sigma F_x = F_{x_1} + F_{x_2} + F_{x_3} \text{ ----- } \text{etc}$$
$$\Sigma F_y = F_{y_1} + F_{y_2} + F_{y_3} \text{ ----- } \text{etc}$$

Ex The forces F_1, F_2 and F_3, all of which act on point A of the bracket are specified in three different ways. Determine the x and y scalar components of each of the three forces.

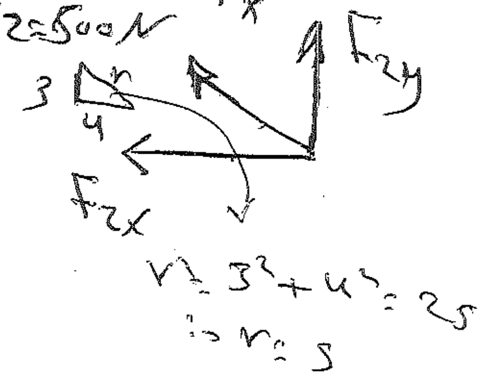


Sol



$F_{1x} = 600\cos 35 = 491\,N$

$F_{1y} = 600\sin 35 = 344\,N$



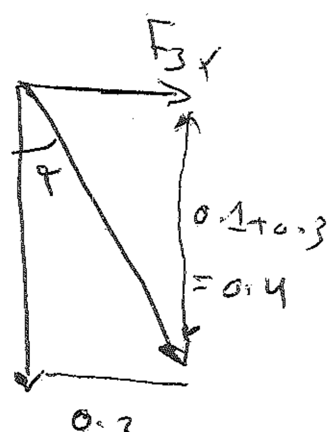
$F_{2x} = -500\,\frac{4}{5} = -400\,N$

$F_{2y} = 500\,\frac{3}{5} = 300\,N$

$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$

$F_{3x} = F_3\sin\alpha = 800\sin 26.6 = 358\,N$

$F_{3y} = -F_3\cos\alpha = -800\cos 26.6 = -716\,N$



Alternatively, the scalar components of F_3 can be obtained by writing F_3 as a magnitude times a unit vector n_{AB} in the direction of the line segment AB.

$$F_3 = F_3\, n_{AB} = 800\left[\frac{0.2i - 0.4j}{\sqrt{(0.2)^2+(-0.4)^2}}\right]$$

$$= 800\,[0.447\, i - 0.894\, j]$$

$$= 358i - 716j \text{ N}$$

$$\therefore F_{3x} = 358 \text{ N}, \quad F_{3y} = -716 \text{ N}$$

Ex

The 500 N force F is applied to the vertical pole as shown in figure below. ① write F in terms of unit vectors i and j and identify both its vector and scalar components ② Determine the scalar components of the force vector F along the x̄ and ȳ ③ Determine the scalar components of F along the x- and y- direction

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Sol part 1

From the ~~figure~~ Fig. a we may write F as

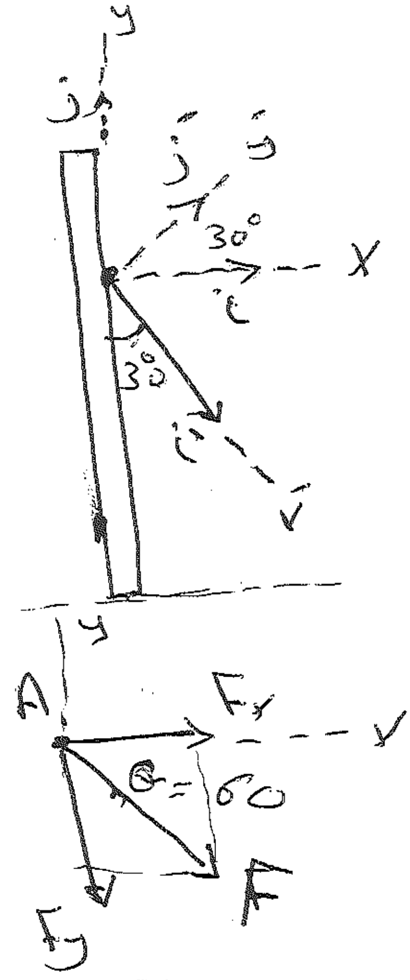


Fig. a

$$F = (F\cos\theta)i - (F\sin\theta)j$$

$$F = (500\cos 60°)i - (500\sin 60°)j$$

$$= (250i - 433j)\ N$$

∴ The scalar components are

$F_x = 250\ N$, $F_y = -433\ N$

The vector components are

$F_x = 250\,i\ N$, $F_y = -433\,j\ N$

2

From the Fig. b we may write F as $F = 500\,i'\ N$ so that the required scalar component are

$F_{x'} = 500\ N$, $F_{y'} = 0$

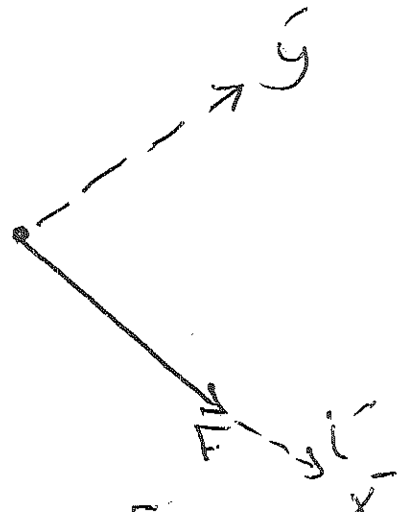


Fig. b

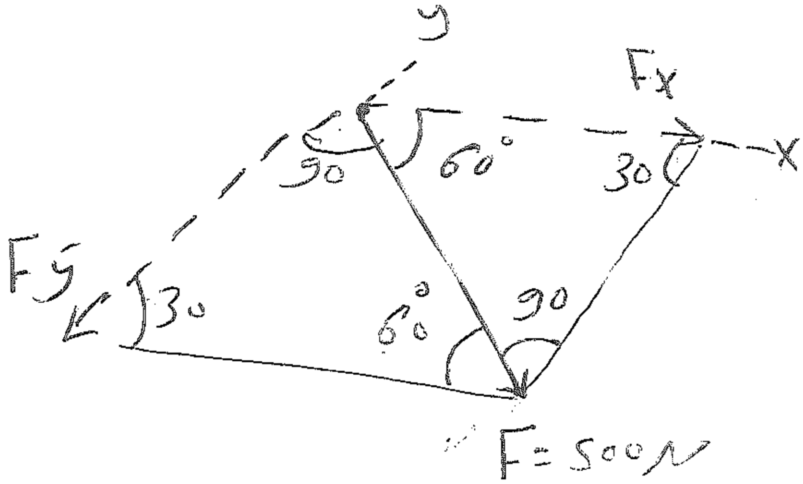
(3) The components of F in the x- and y- directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig c. The magnitudes of the component may be calculated by the law of sines. Thus

$$\frac{|F_x|}{\sin 90} = \frac{500}{\sin 30^\circ}$$

$|F_x| = 1000\,N$

$$\frac{|F_y|}{\sin 60^\circ} = \frac{500}{\sin 30}$$

$|F_y| = 866\,N$



The required scalar component are then

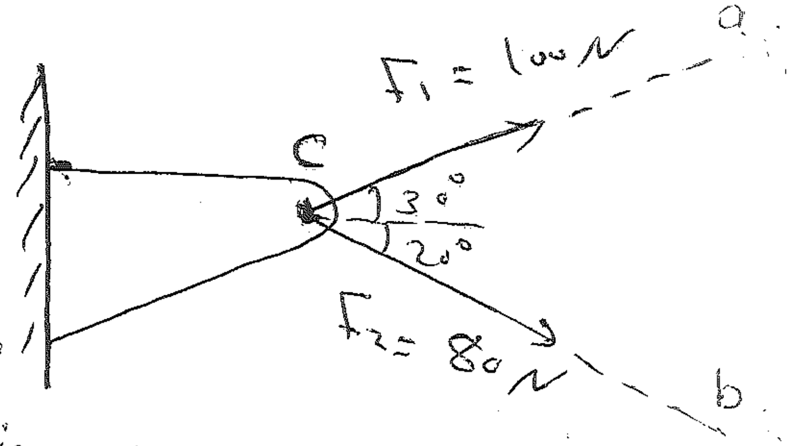
$F_x = 1000\,N \qquad F_y = -866\,N$

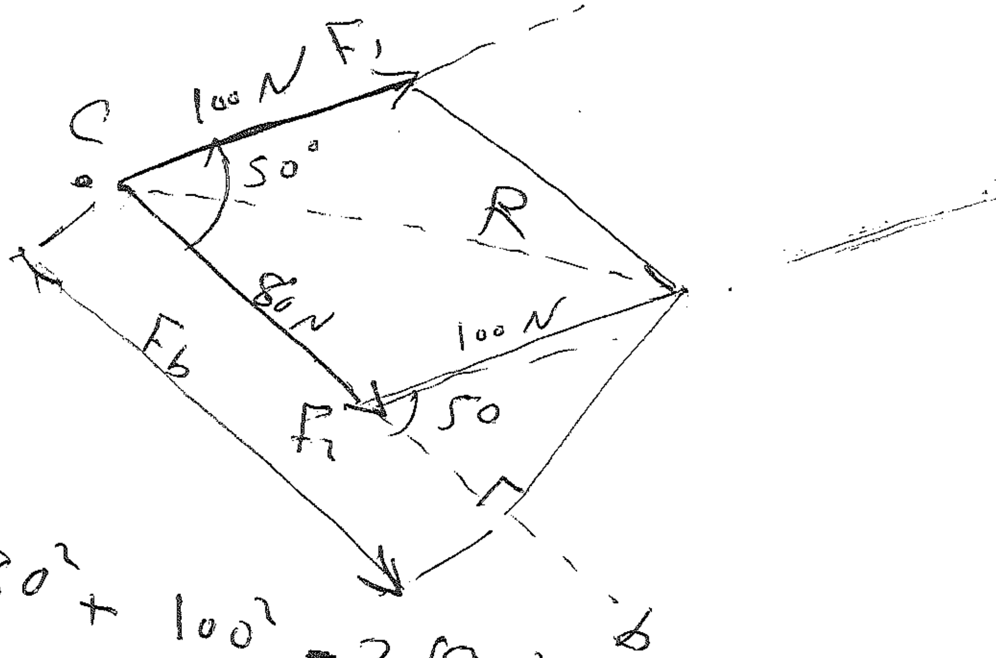
Ex

Forces F_1 and F_2 act on the bracket as shown. Determine the projection F_b of their resultant R onto the b-axis.

Solution

The parallelogram addition of F_1 and F_2 is shown in figure using the law of cosines given as





$$R^2 = 80^2 + 100^2 - 2(80)(100)\cos 130^\circ$$

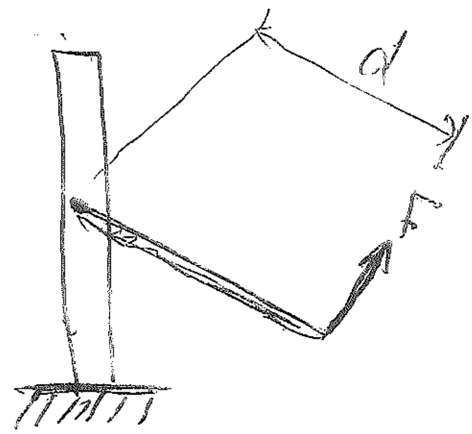
$$R = 163.4\ N$$

The figure also shows the orthogonal projection F_b of R onto the b axis. Its length is

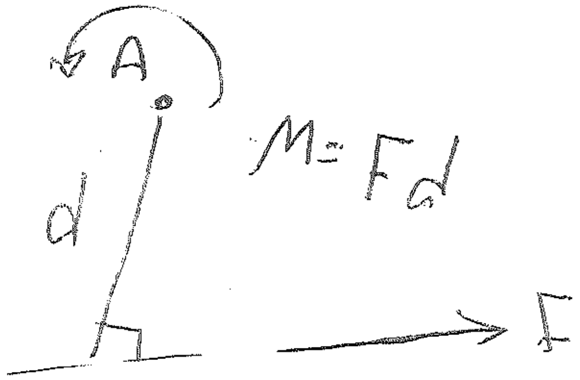
$$F_b = 80 + 100\cos 50 = 144.3\ N$$

Moment

$M = F \cdot d$



M: moment
F: Force
d: displacement



The cross product

$$\vec{M} = \vec{r} \times \vec{F}$$

r: is position vector which runs from the moment reference point A to any point on the line of action of F. The magnitude of this expression is given by:

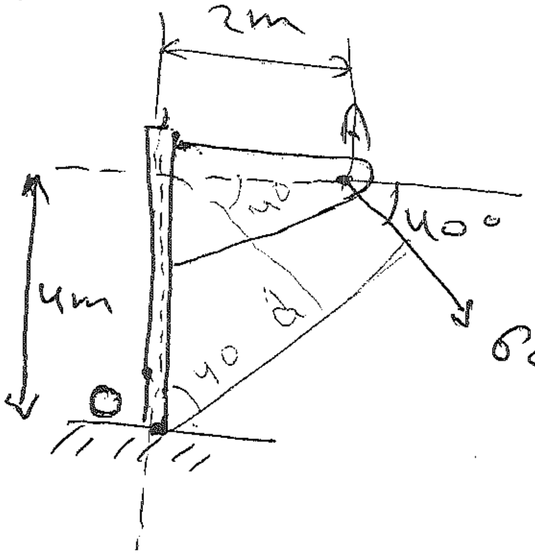
$$M = F r \sin\alpha = F d$$

(20)

Ex

Calculate the magnitude of the moment about base point O of the 600 N force in five different ways.

Sol



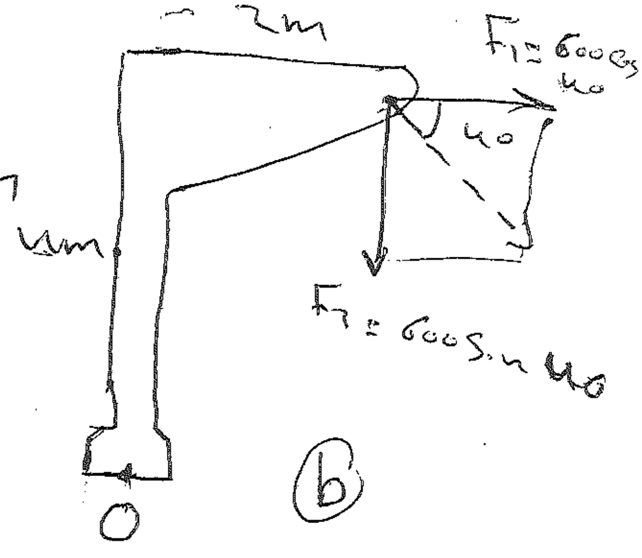
I The moment arm to the 600 N force is

$d = 4\cos 40^\circ + 2\sin 40$

$= 4.35\,m$

① $M = Fd$

$M_O = 600(4.35) = 2610\ N.m$

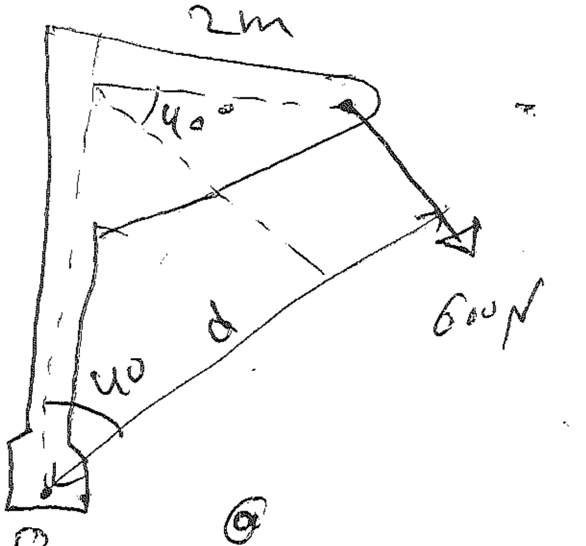


II Replace the force by it's rectangular components at A

$F_1 = 600\cos 40 = 460\,N$

$F_2 = 600\sin 40 = 386\,N$

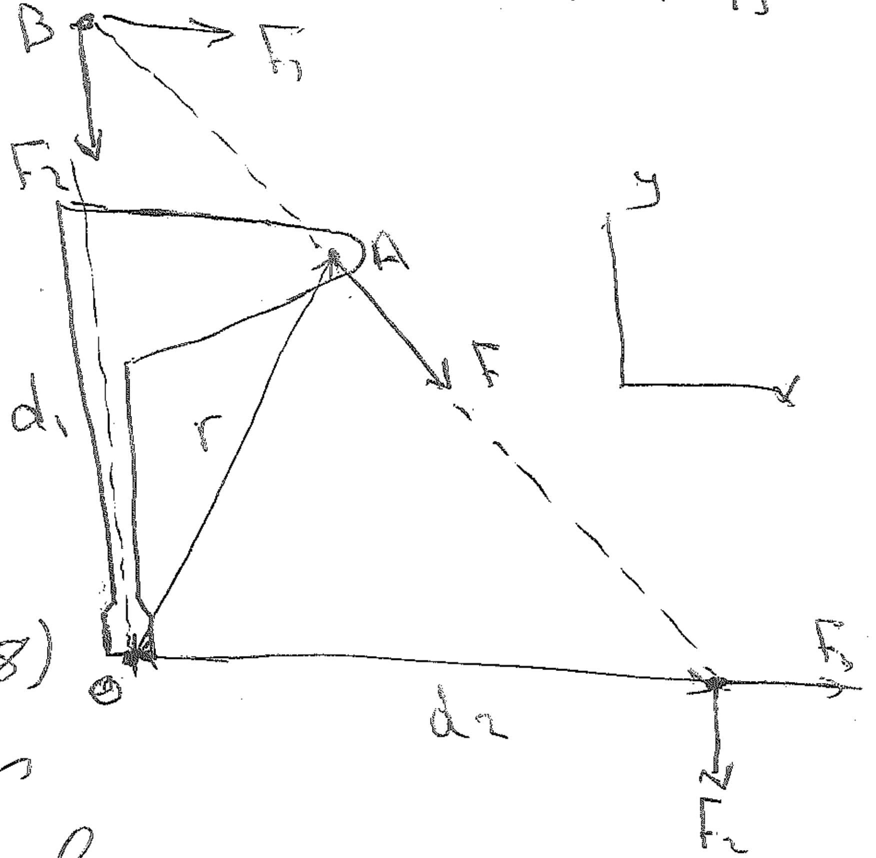
By Vargnon's theorem the moment becames



② $M_O = 460(4) + 386(2)$

$= 2610\ N.m$

By the principle of transmissibility, move the 600 N force along its line of action to point B which eliminates the moment of the component F_2. The moment arm of F_1 becomes



$d_1 = 4 + 2\tan 40$

we obtained d_2

$d_2 = 6.77$ m

$d_1 = 4 + 2\tan 40$
$= 5.68$ m

$\therefore$ $M_O = 460(5.68)$
$= 2610$ N·m

IV/ Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$d_2 = 2 + 4\cot 40° = 6.77$ m

$\therefore$ $M_O = 386(6.77) = 2610$ N

(V) By the vector expression for a moment and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products we have

$$M_o = r \times F = (2i + 4j) \times 600(i \cos 40 - j \sin 40^\circ)$$

$$= -2610\,k \; N.m$$

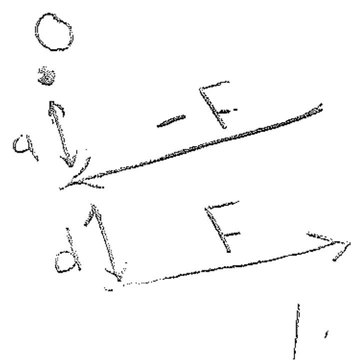
The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression ⇩

$$M_o = 2610 \; N.m$$

Couple

The moment produced by two equal, opposite, and noncollinear forces is called couple. Couples have certain unique properties and have important applications in mechanics.

$$M = F(a+d) - Fa$$
$$= \cancel{F}a + Fd - \cancel{Fa}$$
$$= Fd$$

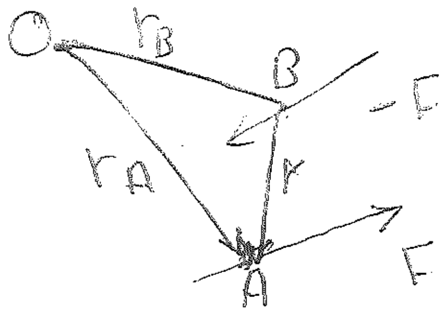


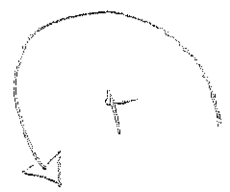
Vector Algebra Method

$$M = r_A \times F + r_B \times (-F)$$
$$= (r_A \times F) - (r_B \times F)$$
$$= (r_A - r_B) F$$

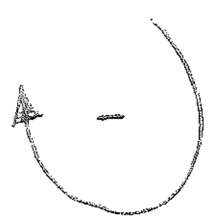
where $r_A - r_B = r$

so $M = r \times F \Rightarrow M = r \times F$



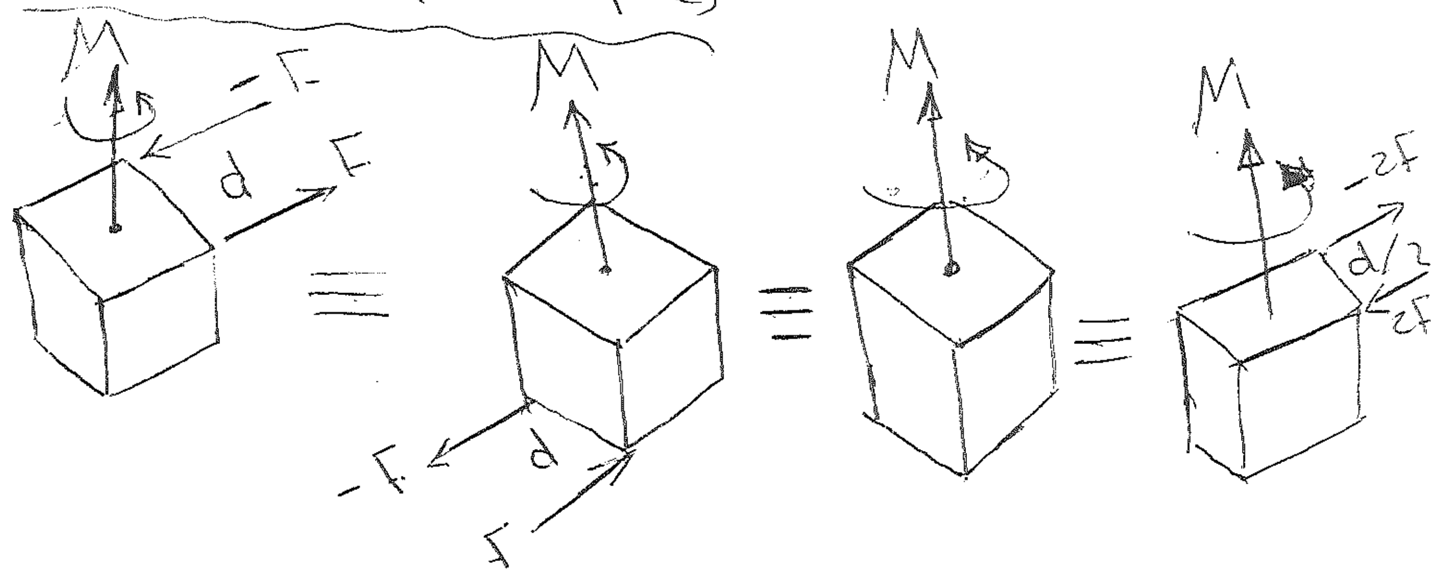


Counterclockwise Couple

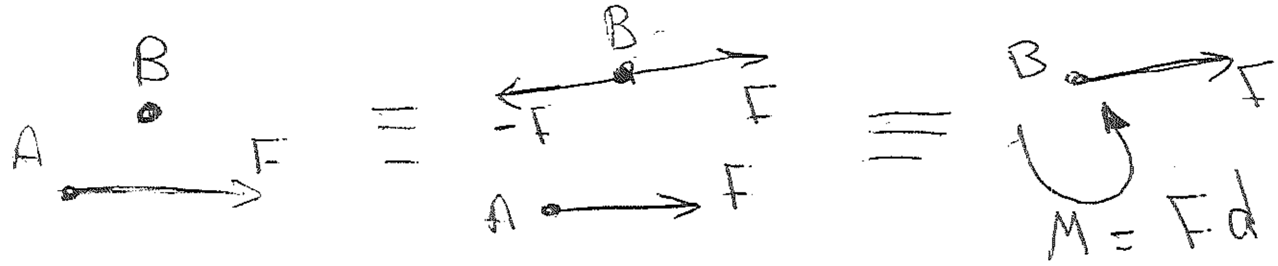


Clockwise couple

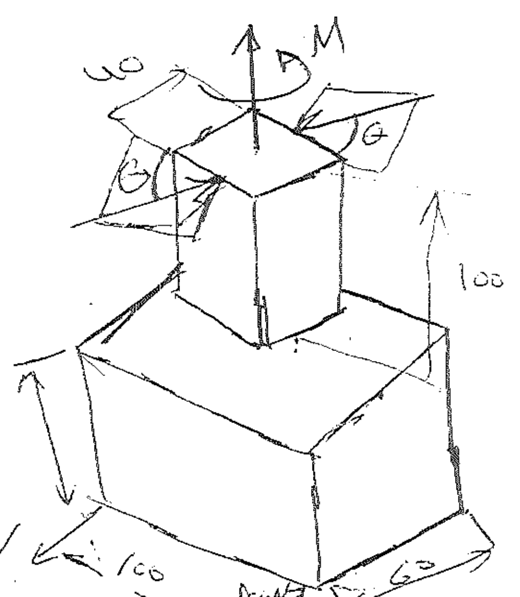
Equivalent couples



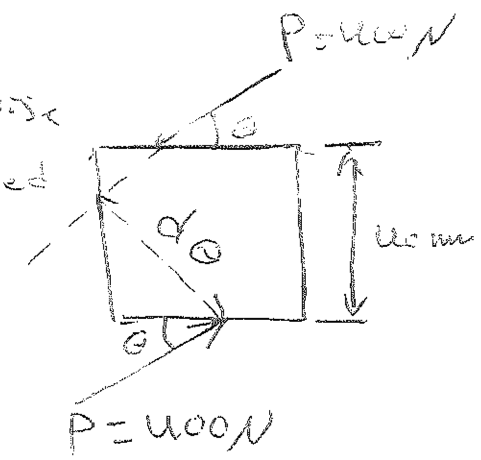
Force - Couple System



EX The rigid structural member is subjected to a couple consisting of the two 100 N forces. Replace this couple by an equivalent couple consisting of the two forces P and -P, each of which has a magnitude of 400 N. Determine the proper angle θ.



Solution



The original ~~force~~ Couple is counterclockwise when the plane of the forces is viewed from above, and its magni

$$M = F d = 100 \times 0.1 = 10 \ N.m$$

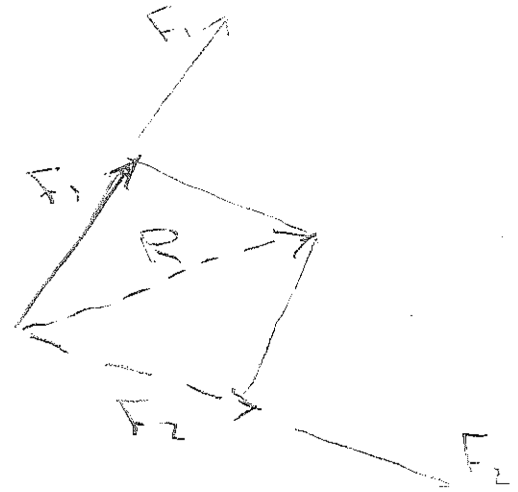
The forces P and -P produce a counterclockwise Couple

$$M = 400\,(40/1000)\,\cos\theta$$

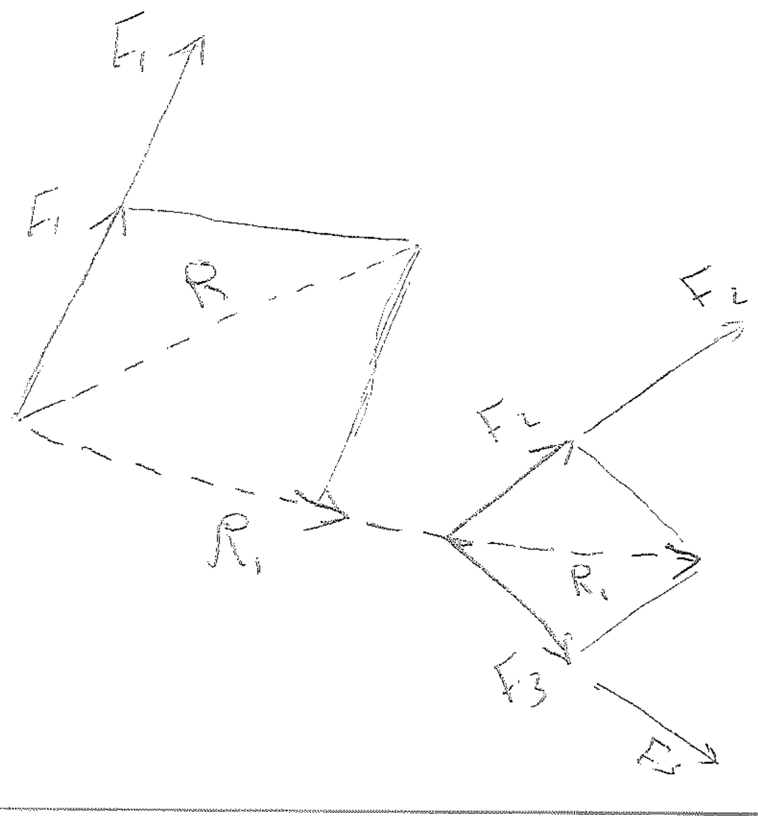
$$M = 400 \times 0.04 \cos\theta$$

$$\theta = \cos^{-1}\frac{10}{16} = 51.3^{\circ}$$

Resultants



$$R = F_1 + F_2 = \Sigma F$$



$$R = F_1 + F_2 + F_3 + \text{---} = \Sigma F$$

$$R_x = \Sigma F_x$$

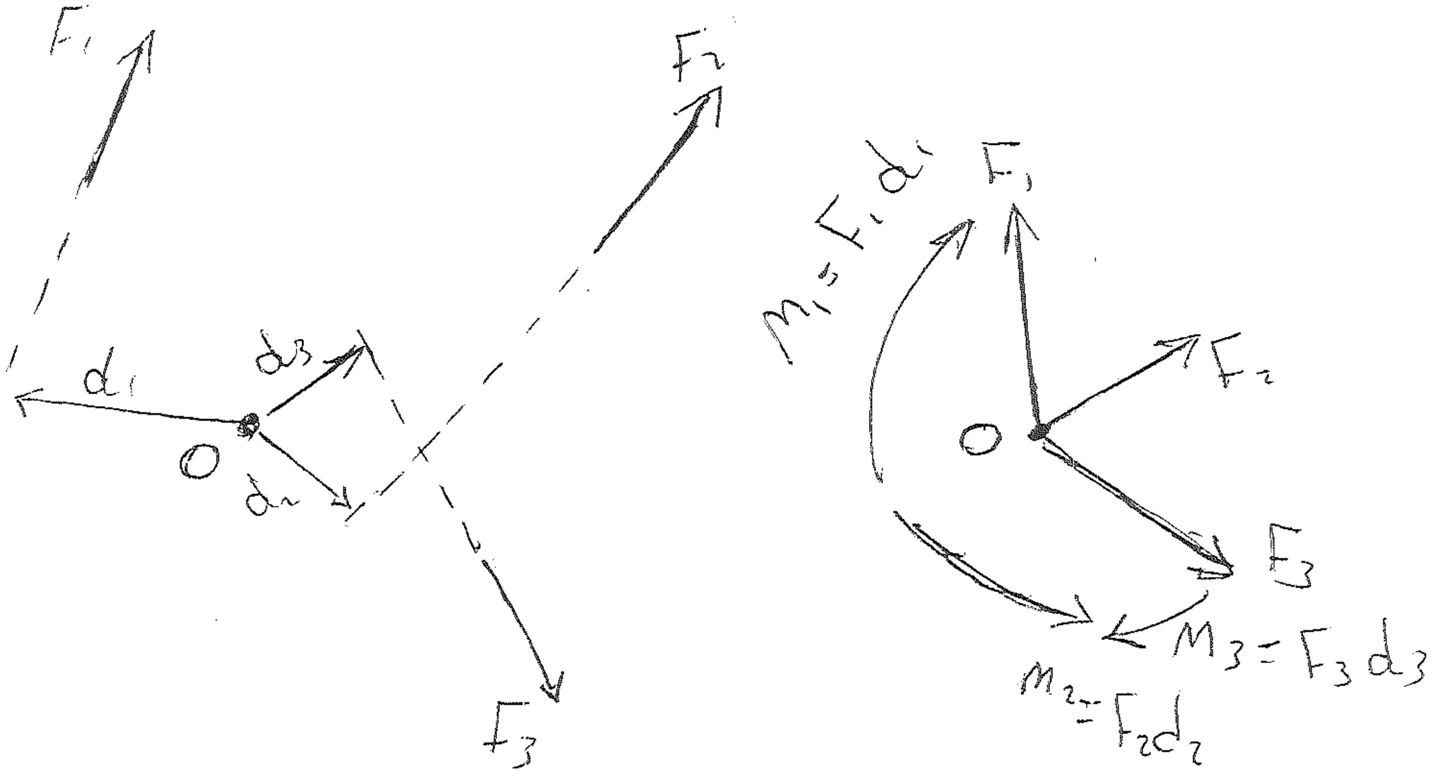
$$R_y = \Sigma F_y$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

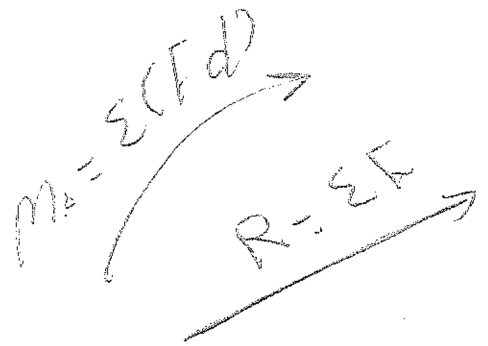
Algebraic Method

1. Choose a convenient reference point and move all forces to that point. This process is depicated for a three-force system as shown in figs below:-



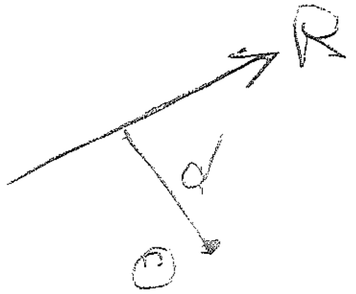
M_1, M_2 and M_3 are the couples resulting from the transfer of forces F_1, F_2 and F_3 from their respective original lines of action to lines of action through point O.

② Add all forces at O to form the resultant force R and add all couples to form the resultant couple M_o. we now have the single force - couple system as shown in fig below



③ In Fig below find the line of action of R by requiring R to have a moment of M_o about O.

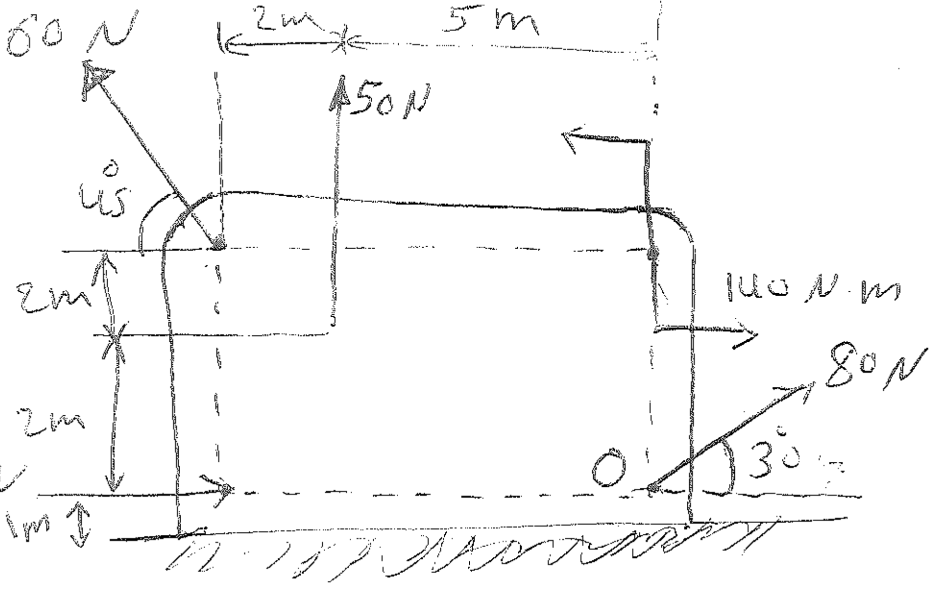
$$d = \frac{M_o}{R}$$



principle of moments

$$R = \Sigma F$$
$$M_o = \Sigma M = \Sigma(Fd)$$
$$Rd = M_o$$

EX Determine the resultant of the four and one couple which act on the plate shown



(28)

Sol point O is selected as a convenient reference point for the force couple system that is to represent the given system.

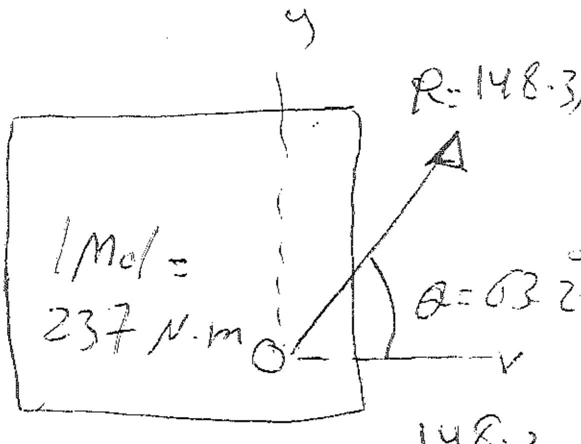
$$R_x = \Sigma F_x \Rightarrow R_x = 40 + 80\cos 30 - 60\cos 45$$
$$= 66.9\,N$$

$$R_y = \Sigma F_y \Rightarrow R_y = 50 + 80\sin 30 + 60\cos 45$$
$$= 132.4\,N$$

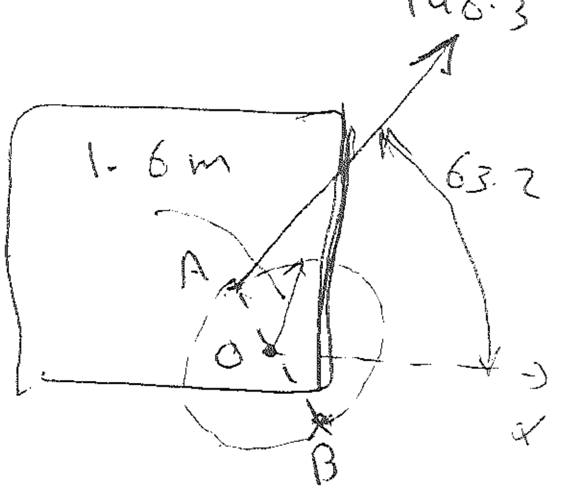
$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(66.9)^2 + (132.4)^2} = \boxed{148.3\,N}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{132.4}{66.9} = \boxed{63.2^\circ}$$

$$M_O = \Sigma(Fd)$$
$$= 140 - 50(5) + 60\cos 45^\circ(4) - 60\sin 45^\circ(7)$$
$$= -237\ N\cdot m$$



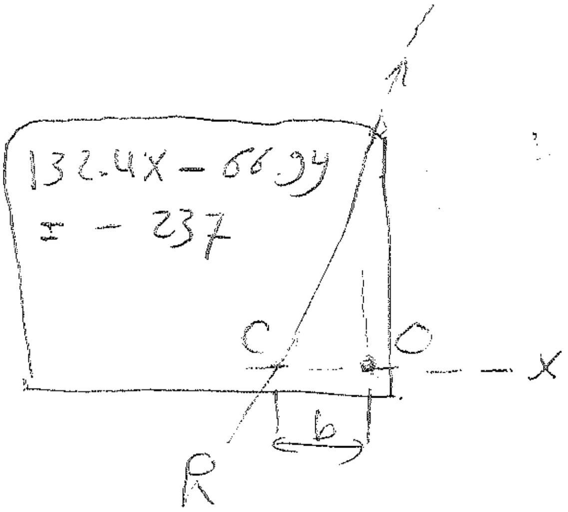
$$Rd = |M_O|$$
$$148.3d = 237$$
$$d = \boxed{1.6\ m}$$



The resultant R may also be calculated by determining its intercept distance b to point C on the x-axis - with R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O|$$

$$b = \frac{237}{132.4} = 1.792\,m$$



$$r \times R = M_O$$

$$(x\mathbf{i} + y\mathbf{j}) \times (-66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

so $\boxed{132.4x - 66.9y = -237}$

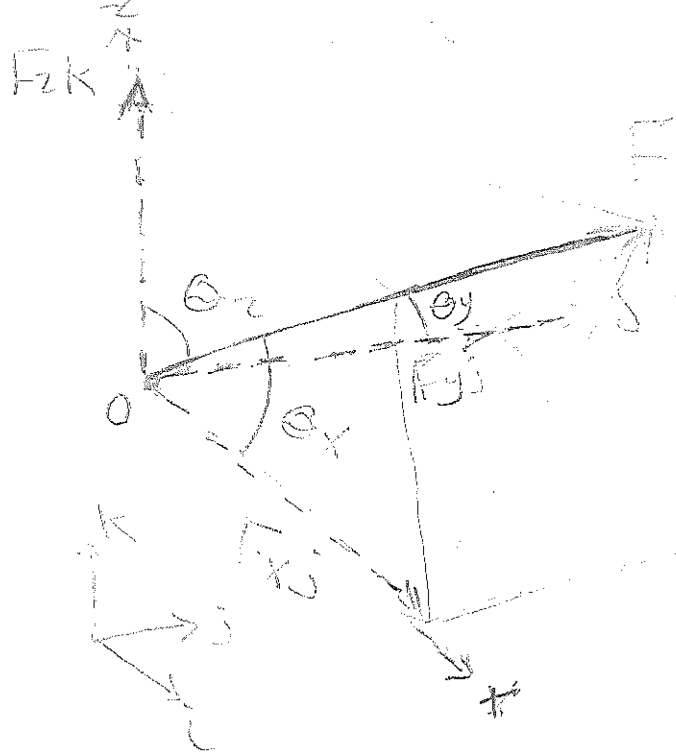
by letting $y = 0$

so $\boxed{x = -1.792\,m}$

Three - Dimensional Force System

Rectangular Component

Many problems in mechanics require analysis in three dimensions.



$$\text{Force } (F) \begin{cases} F_x \\ F_y \\ F_z \end{cases}$$

$$\boxed{\begin{aligned} F_x &= F\cos\theta_x \\ F_y &= F\cos\theta_y \\ F_z &= F\cos\theta_z \end{aligned}}$$

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = F_x i + F_y j + F_z k$$

$$F = F(i\cos\theta_x + j\cos\theta_y + k\cos\theta_z)$$

i, j and k are unit vector in x-, y- and z- directions, respectively. Using the direction cosines of F which are $l = \cos\theta_x$, $m = \cos\theta_y$ and $n = \cos\theta_z$ where $l^2 + m^2 + n^2 = 1$ we may write the force as

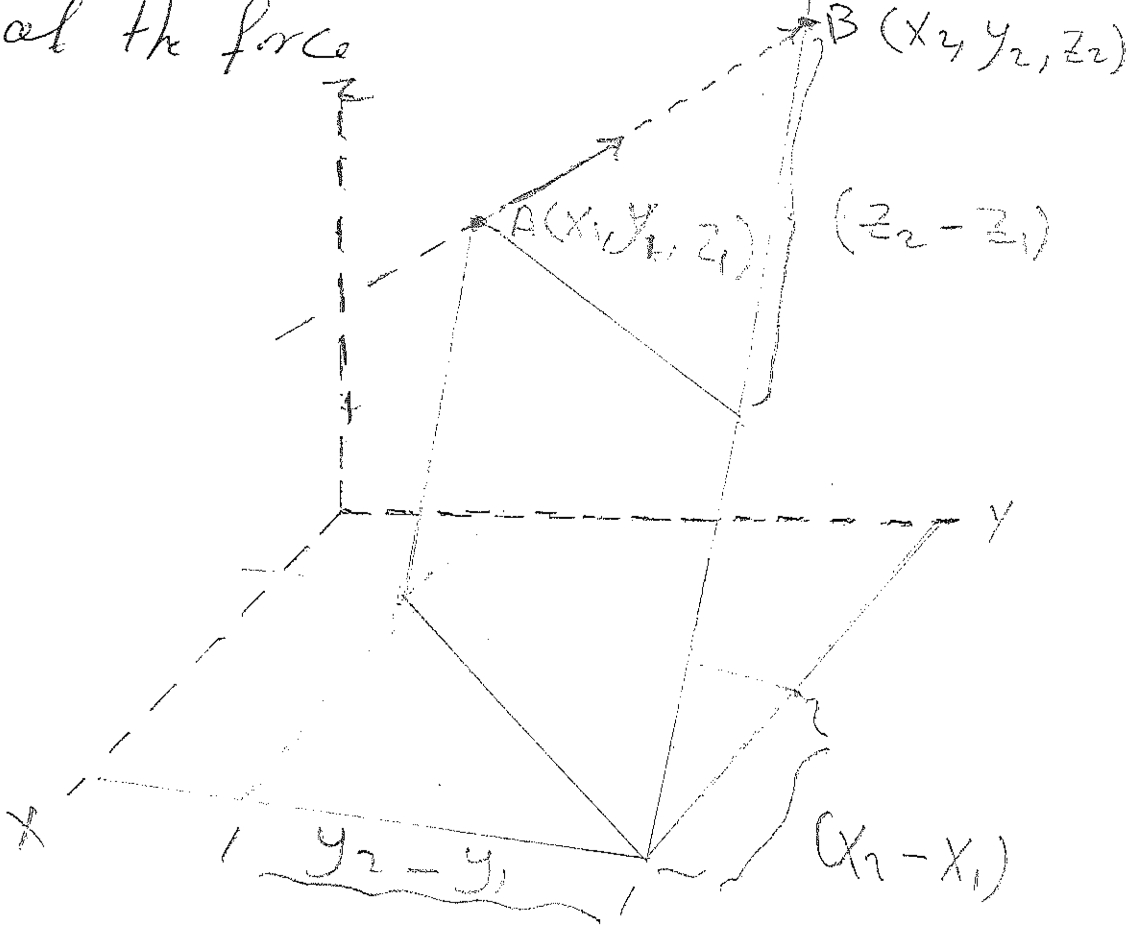
$$\boxed{\mathbf{F} = F(li + mj + nk)}$$

where n_F is unit vector

$$n_F = (li + mj + nk)$$

$$\mathbf{F} = F n_F$$

(a) Specification by two points on the line of action of the force



$$\mathbf{F} = F n_F = F \frac{\overrightarrow{AB}}{\overline{AB}} = F \frac{(x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

x, y, and z scalar component of $\mathbf{F}$ are the scalar coefficients of the unit vectors i, j and k respectively.

(b) Specification by two angles which orient the line of action of the force

$$F_{xy} = F \cos\phi$$

$$F_z = F \sin\phi$$

horizontal component F_{xy} into x- and y-components

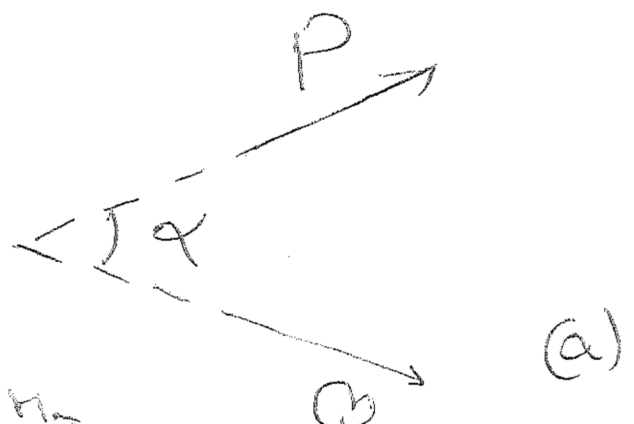
$$F_x = F_{xy} \cos\theta$$
$$= F\cos\phi \cos\theta$$

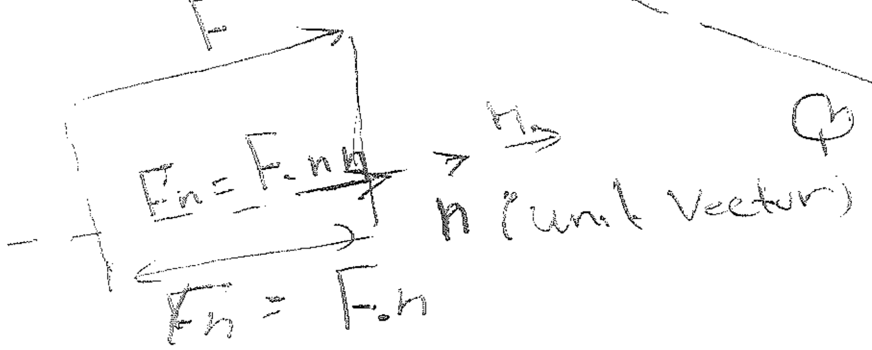
$$F_y = F_{xy} \sin\theta$$
$$= F\cos\phi \sin\theta$$

F_x, F_y and F_z are scalar components of **F**.

Dot product

$$P \cdot Q = PQ \cos\alpha$$





(b)

If n is a unit vector in specified direction, the projection F in the n-direction Fig (b) has the magnitude

$F_n = F \cdot n$

$F \cdot n$ by unit vector n give

$F_n = (F \cdot n)\, n = F \cdot n n$

[Direction cosines of n are α, β and γ

$n = \alpha i + \beta j + \gamma k$

$F_n = F \cdot n = F(li + mj + nk) \cdot (\alpha i + \beta j + \gamma k)$

$= F(l\alpha + m\beta + n\gamma)$

$i \cdot i = 1 = j \cdot j = 1 = k \cdot k = 1$

and $i \cdot j = j \cdot i = i \cdot k = k \cdot i = j \cdot k$

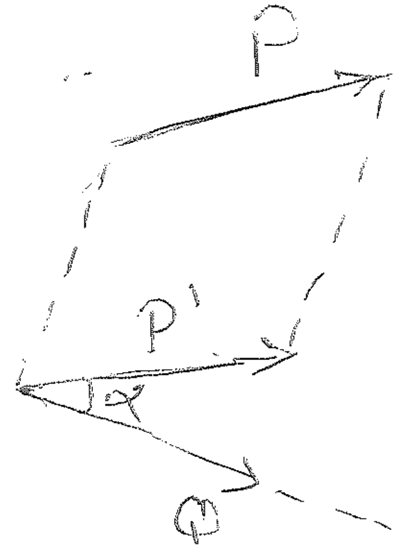
$= k \cdot j = 0$

i, j and k have unit length and are mutually perpendicular

Angle between two Vector

$$\theta = \cos^{-1} \frac{F \cdot n}{F}$$

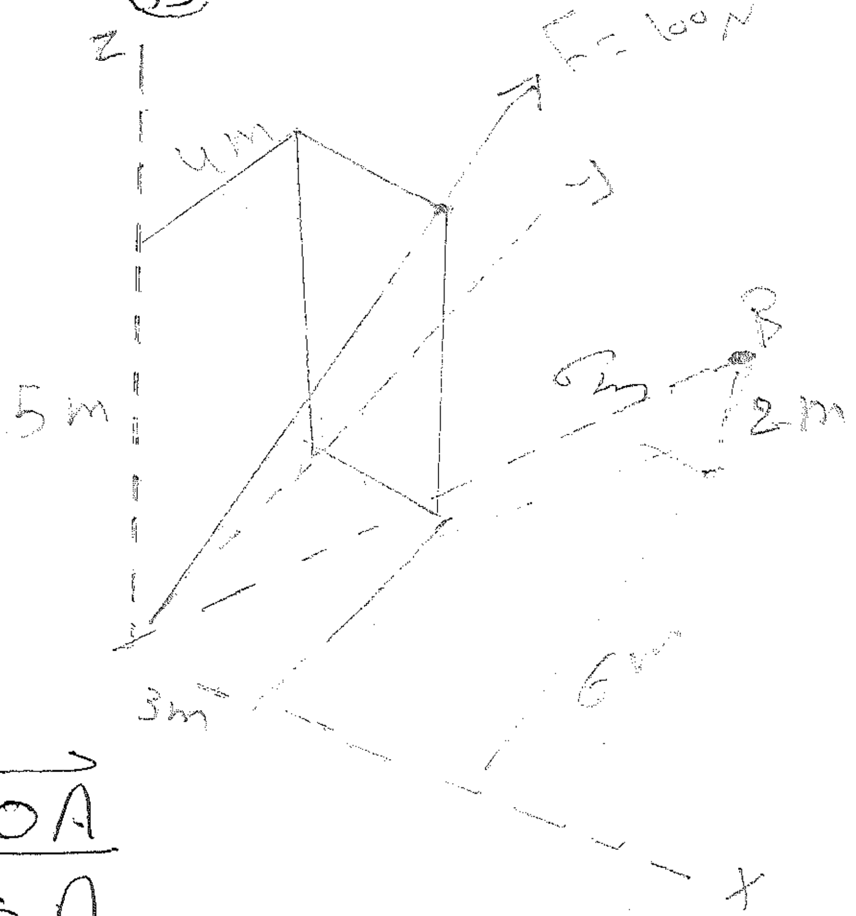
$$\theta = \cos^{-1} \frac{P \cdot Q}{P Q}$$



Ex

A Force with magnitude of 100 N is applied at the origin O of the axes x-y-z as shown. The line of action of F passes through the point A whose coordinates are 3 m, 4 m and 5 m. Determine (a) the x, y and z scalar component of F, (b) the projection F_{xy} of F on the x-y plane, and (c) the projection F_{OB} of F along the line OB.

(35)



Sol

(a)

$$F = F\,n_{OA} = F\,\frac{\overrightarrow{OA}}{OA}$$

$$= 100\left[\frac{3i+4j+5k}{\sqrt{3^2+4^2+5^2}}\right]$$

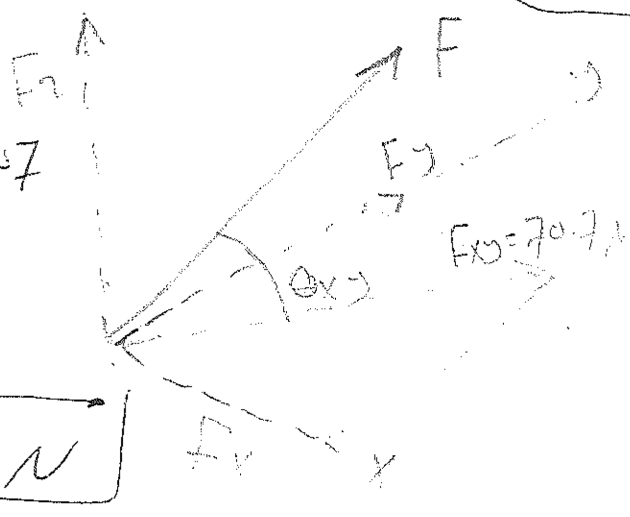
$$= 100\,[0.424i + 0.566j + 0.707k]$$

$$= 42.4i + 56.6j + 70.7k \text{ N}$$

(1)

$\therefore$ $\boxed{F_x = 42.4 \text{ N}}$ $\boxed{F_y = 56.6 \text{ N}}$ $\boxed{F_z = 70.7 \text{ N}}$

(b)



$$\cos\theta_{xy} = \frac{\sqrt{3^2+4^2}}{\sqrt{3^2+4^2+5^2}} = 0.707$$

$$F_{xy} = F\cos\theta_{xy}$$

$$= 100(0.707) = \boxed{70.7 \text{ N}}$$

36

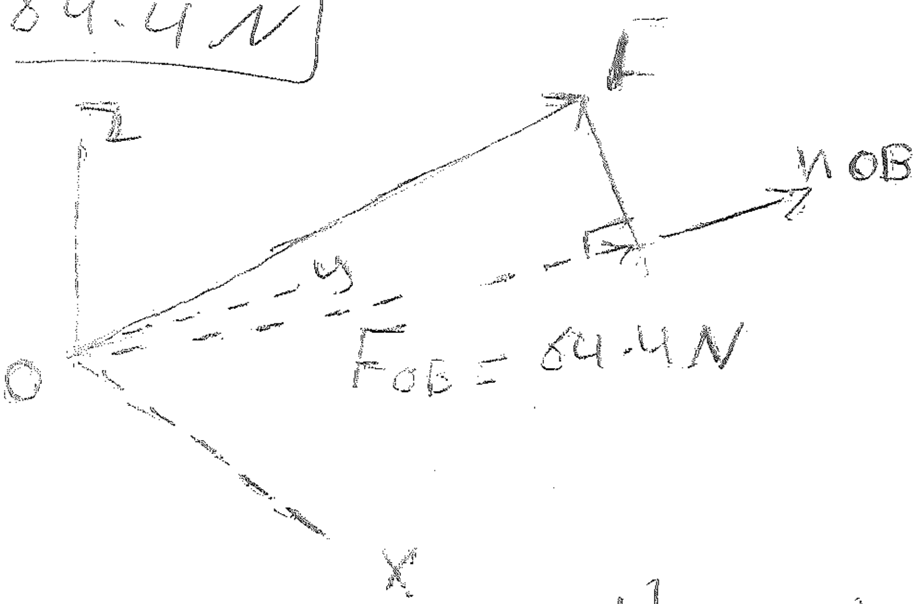
$$n_{OB} = \frac{\overrightarrow{OB}}{OB} = \frac{6i + 6j + 2K}{\sqrt{6^2 + 6^2 + 2^2}} = 0.688i + 0.688j + 0.229k$$

2

$$F_{OB} = F \cdot n_{OB} = (42.4i + 56.6j + 70.7k) \cdot (0.688i + 0.688j + 0.229k)$$

$$= (42.4)(0.688) + (56.6)(0.688) + (70.7)(0.229)$$

$$= \boxed{84.4N}$$



If we wish to express the projection as a vector we write

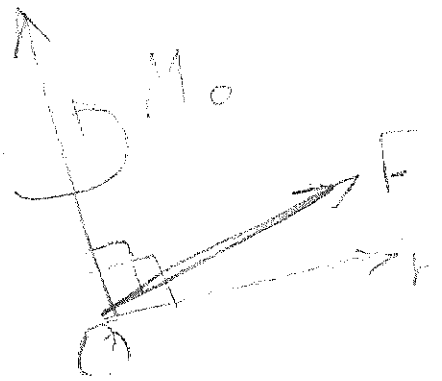
$$F_{OB} = F \cdot n_{OB} \, n_{OB}$$

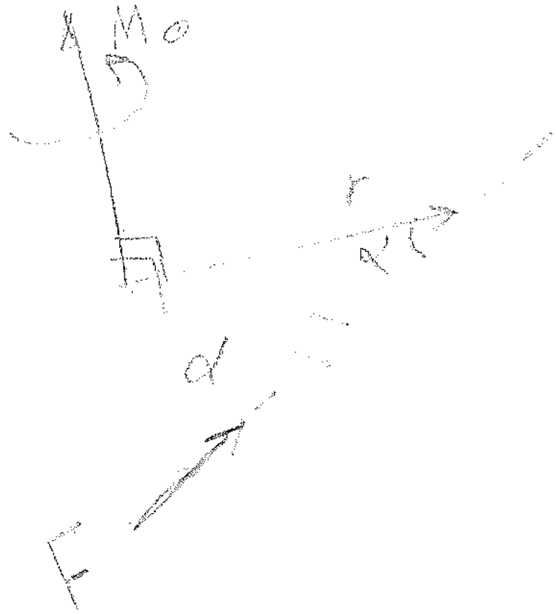
$$= 84.4(0.688i + 0.688j + 0.229k)$$

$$= 58.1i + 58.1j + 19.35 \, KN$$

Moment and Couple

Moment in Three Dimensions



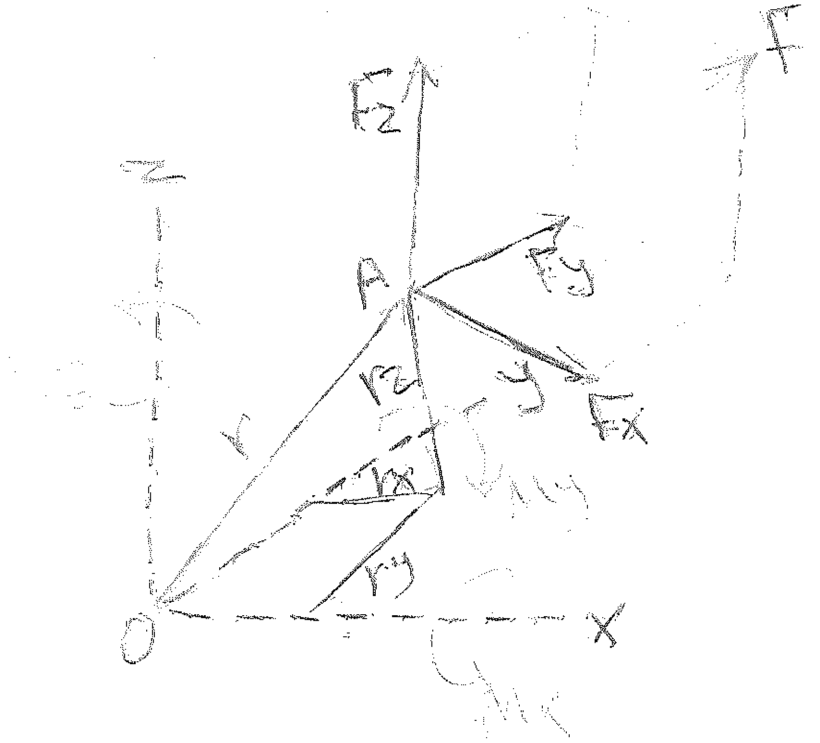


$$\boxed{M_O = r \times F}$$

Evaluating the cross product

$$\boxed{M_O = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}}$$

$$M_O = (r_y F_z - r_z F_y)\, i - (r_z F_x - r_x F_z)\, j + (r_x F_y - r_y F_x)\, K$$

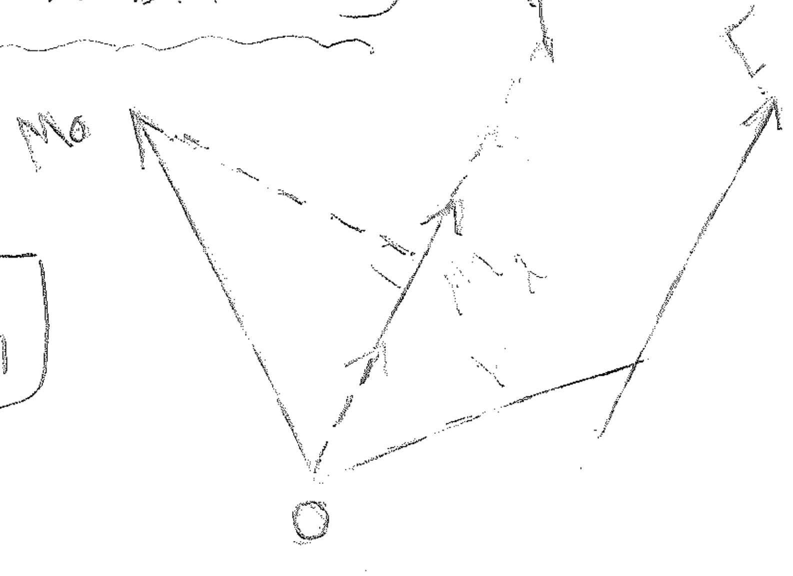


$$M_x = r_y F_z - r_z F_y$$

$$M_y = r_z F_x - r_x F_z$$

$$M_z = r_x F_y - r_y F_x$$

Moment about Arbitarary



$$\boxed{M_\lambda = (r \times F \cdot n)\, n}$$

(39*)

$$|M_\lambda| = M_\lambda = \begin{vmatrix} r_x & r_y & r_z \\ F_x & F_y & F_z \\ \alpha & \beta & \gamma \end{vmatrix}$$

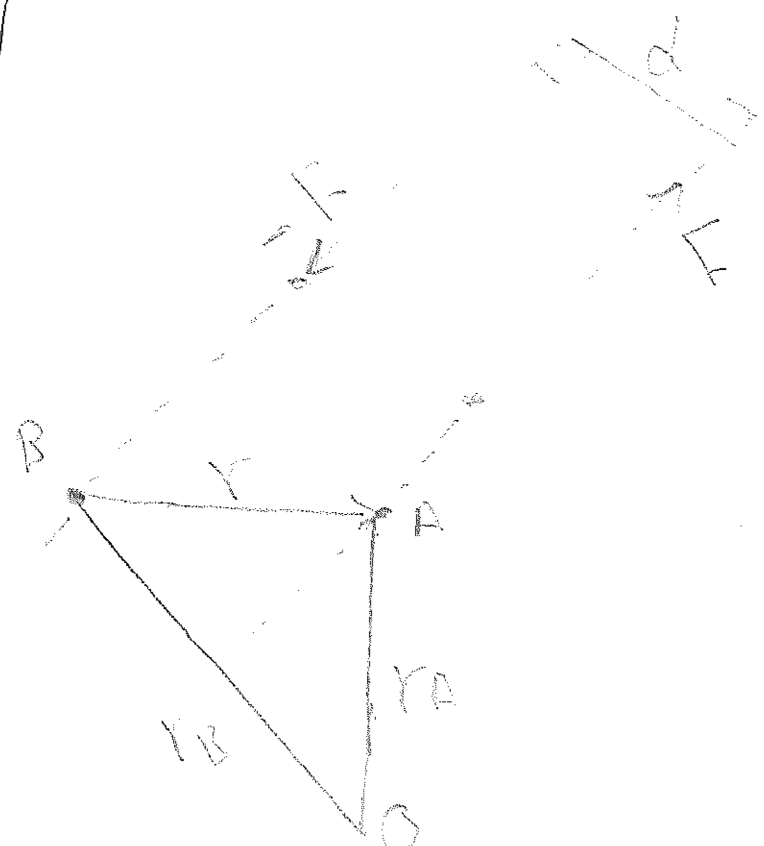
α, β and γ are the direction cosines of the unit vector n

$$\boxed{M_o = \sum (r \times F) = r \times R}$$

↳ Varignon's theorem in three dimensions

Couples in three dimensions

$$\boxed{M = r \times F}$$



Ex A tension T of magnitude 10 kN is applied to the cable attached to the top A of the rigid mast and secured to the ground B. Determine the moment M_z of T about the Z direction Z-axis passing through the base O.

Sol

(a)

$$T = T_{n_{AB}} = 10\left[\frac{12i - 15j + 9k}{\sqrt{(12)^2 + (-15)^2 + 9^2}}\right]$$

$$= 10(0.566i - 0.707j + 0.424k)$$

kN

$$M_o = r \times F$$

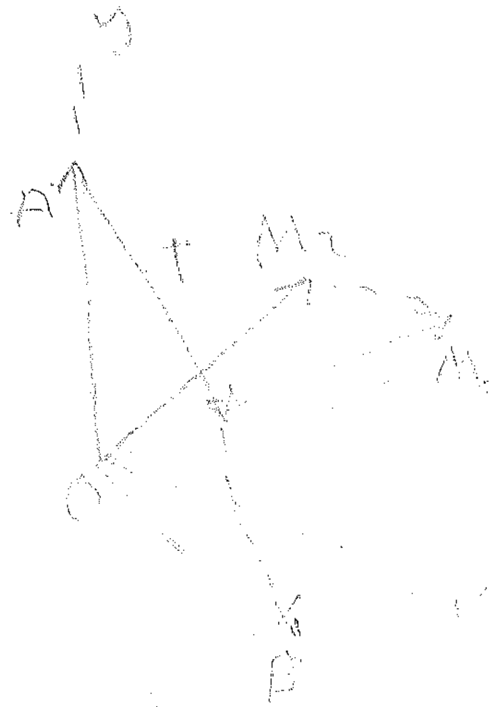
$$M_o = 15j \times 10(0.566i - 0.707j + 0.424k)$$

$$= 150(-0.566k + 0.424i) \text{ kN.m}$$

$$M_z = M_o \cdot k$$

$$M_z = 150(-0.566k + 0.424i) \cdot k$$

$$= \boxed{-84.9 \text{ kN.m}}$$



Sol (b)

$$T_{xy} = \frac{\sqrt{15^2+12^2}}{\sqrt{15^2+12^2+9^2}}$$

$$= 0.906$$

$$T_{xy} = 10(0.906)$$

$$= 9.06 \text{ kN}$$

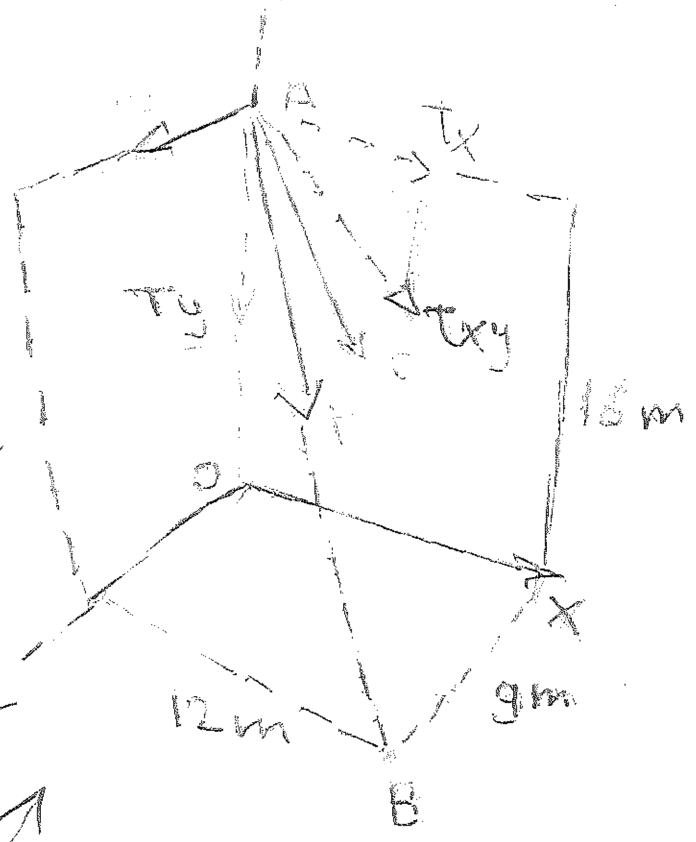
$$M_z = T_{xy}\, d \implies d = \frac{M_z}{T_{xy}}$$

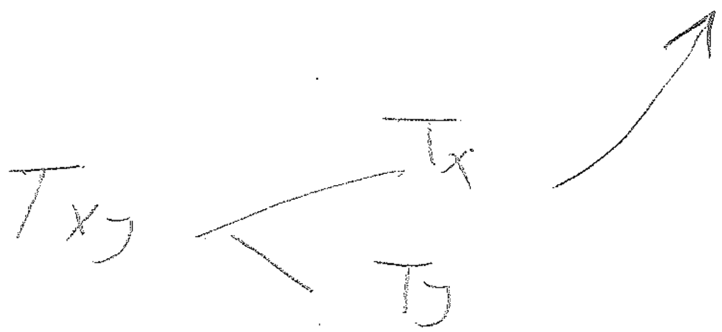
$$d = 15\,\frac{12}{\sqrt{12^2+15^2}} = 9.37 \text{ m}$$

$$M_z = 9.06 \times 9.37 = \boxed{84.9 \text{ kN}}$$

(C)

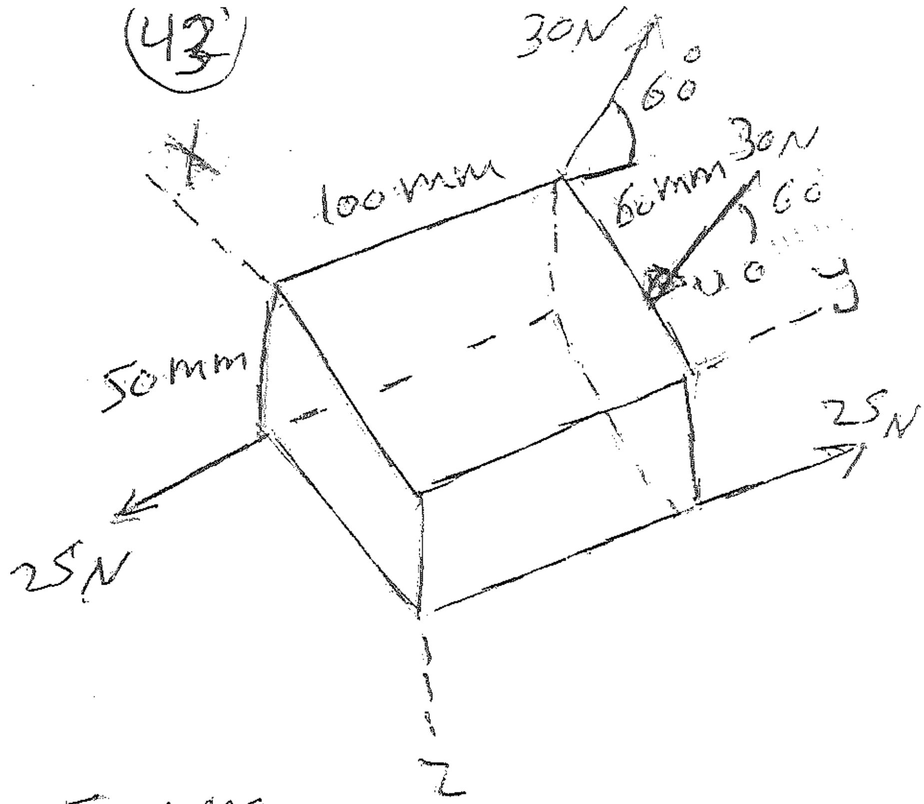
$T_z = 10(0.566)$
$= 5.66$ kN
$M_z = 5.66(15)$
$= \boxed{84.9 \text{ kN}}$



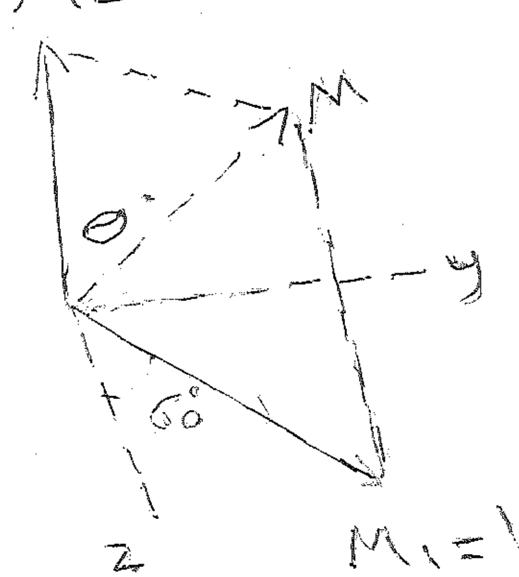


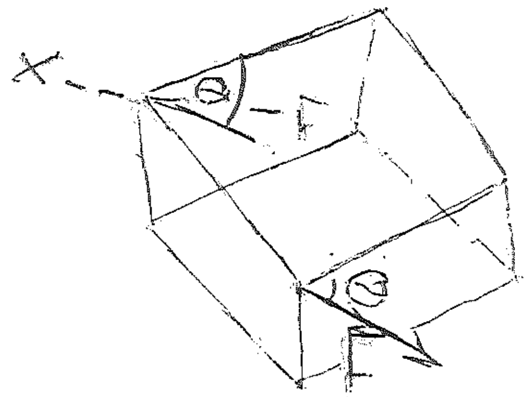
Ex Determine the magnitude and direction of the couple M which will replace the two given couples and still produce the same external effect on the block. Specify the two forces F and -F, applied in the two faces of the block parallel to the y-x plane which may replace the four given forces. The 30-N forces act parallel to the y-z plane.

(43)



Sol





$$M_y = 1.8 \sin 60^\circ = \boxed{1.559\ N.m}$$

$$M_z = -2.50 + 1.8 \cos 60 = \boxed{-1.6\ N.m}$$

$$M = \sqrt{(1.559)^2 + (-1.6)^2} = \boxed{2.23\ N.m}$$

$$\theta = \tan^{-1} \frac{1.559}{1.6} = \tan^{-1} 0.974$$

$$= 44.3^\circ$$

$$M = Fd$$

$$F = \frac{2.23}{0.1} = 22.3$$

Resultant

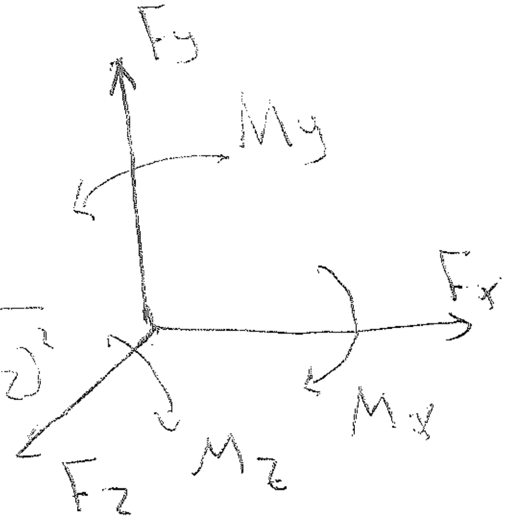
$$R = F_1 + F_2 + F_3 + \cdots = \Sigma F$$

$$M = M_1 + M_2 + M_3 + \cdots = \Sigma M = \Sigma(r \times F)$$

$$R_x = \Sigma F_x, \quad R_y = \Sigma F_y$$

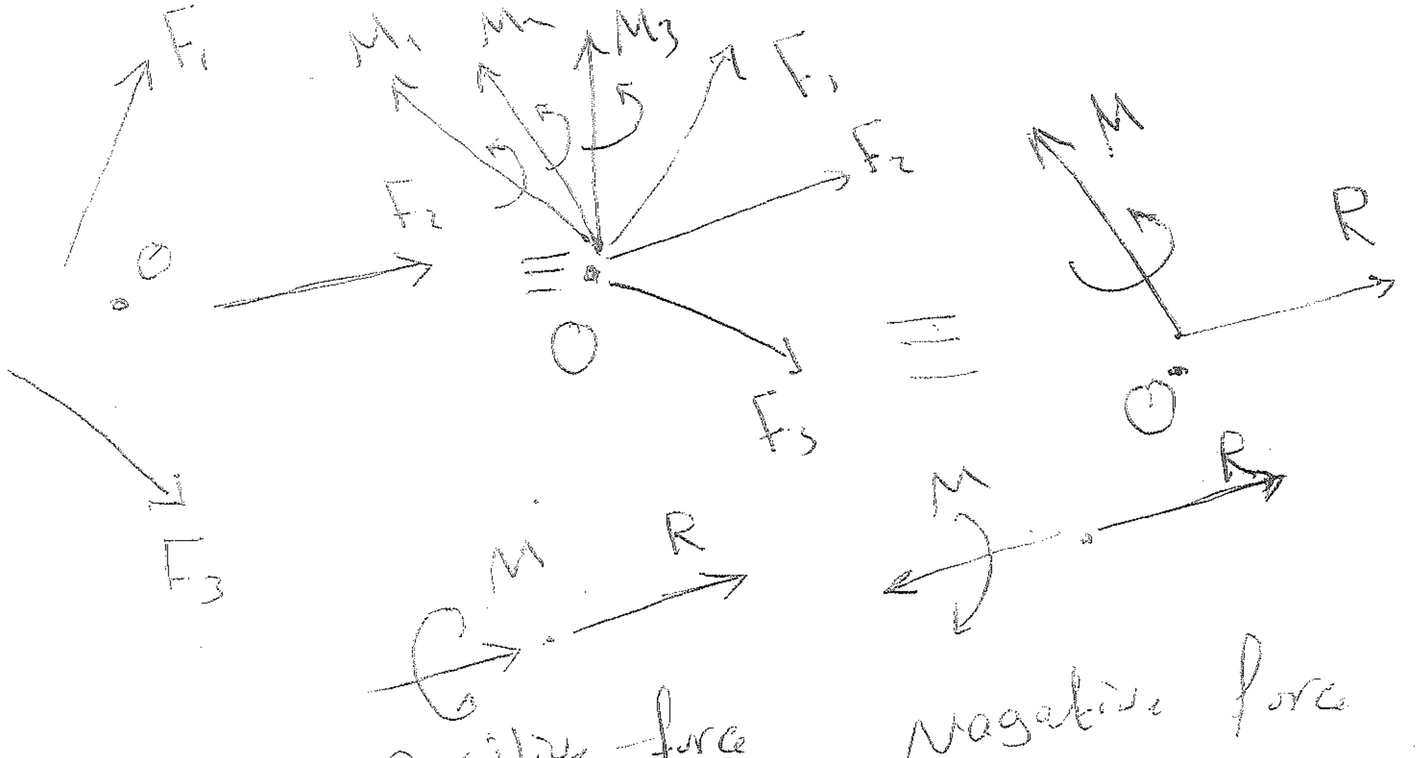
$$R_z = \Sigma F_z$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$



$$M_x = \Sigma(r \times F)_x, \quad M_y = \Sigma(r \times F)_y$$

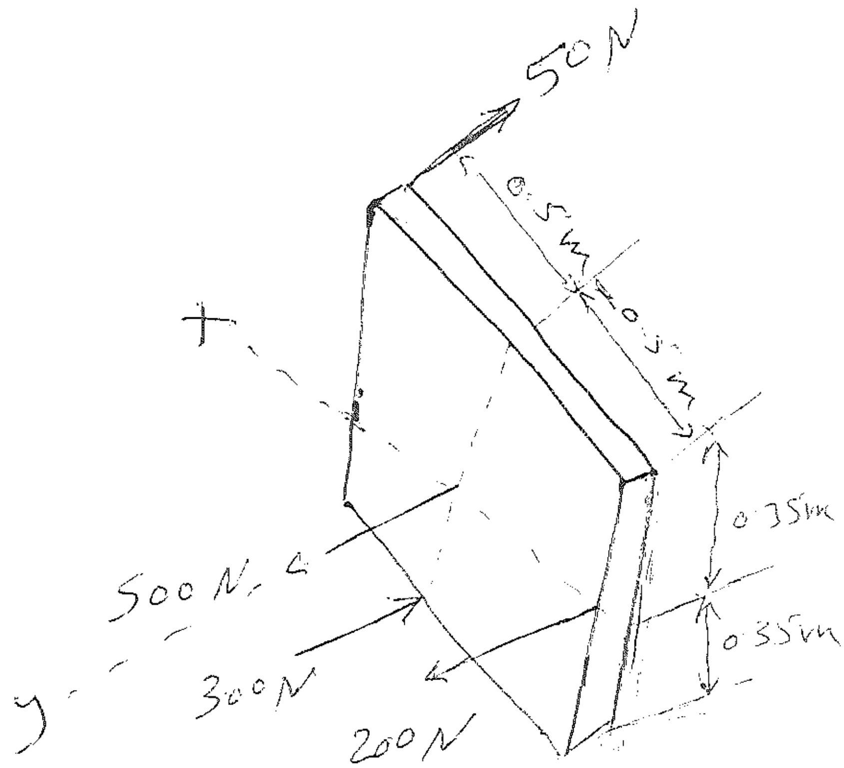
$$M_z = \Sigma(r \times F)_z$$



positive force   Negative force

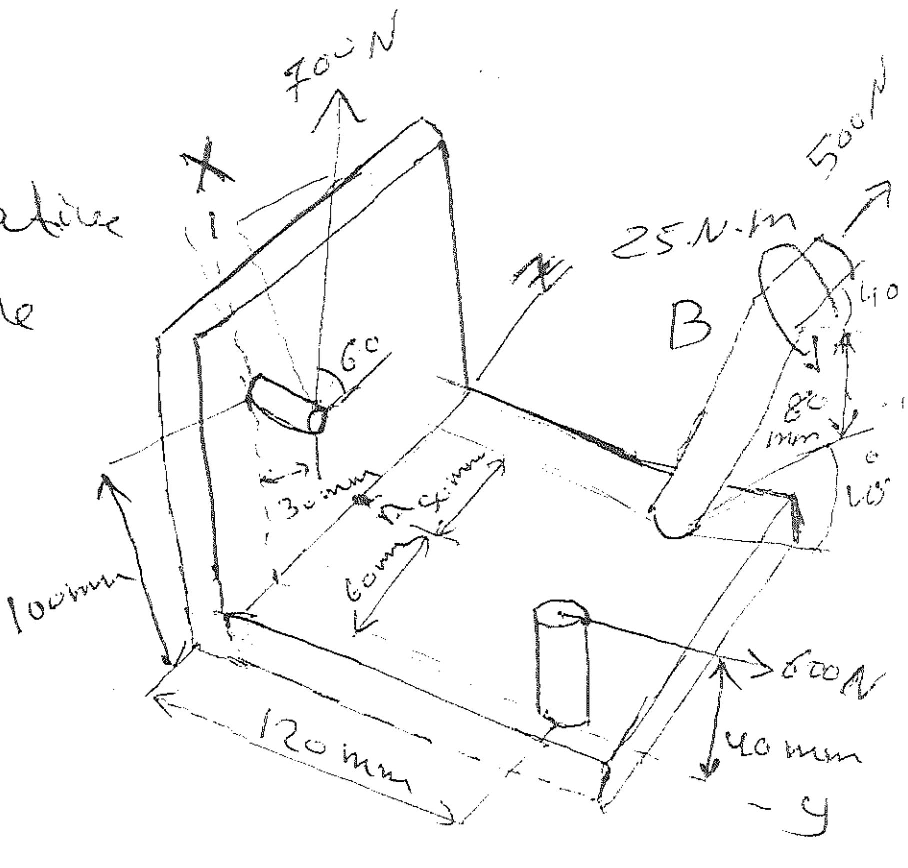
Ex 2.14

Determine the resultant of the system of forces which act on the plate. Solve with a vector approach.



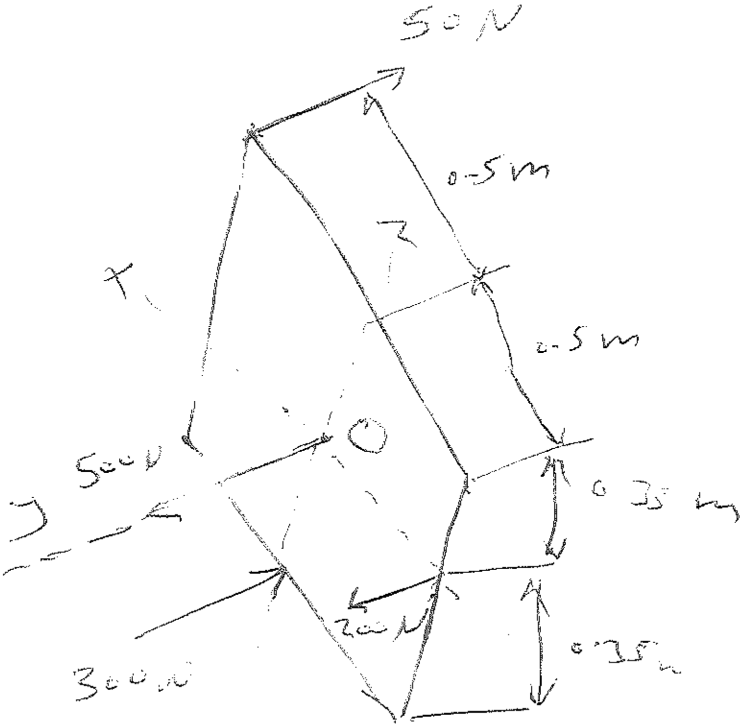
Ex 2.15

Replace the two forces and the negative wrench by a single force R applied at A and the corresponding M.



Solution

EX 2/14



نلاحظ أن جميع القوى باتجاه (y)
لذلك وحدة المحصلة في (j)

$$R = \sum F$$

$$\therefore R = (200 + 500 - 300 - 50) = \boxed{350\,j\ N}$$

$$M_O = [50(0.35) - 300(0.35)]i + [-50(0.5) - 200(0.5)]K$$

$$= \boxed{-87.5j - 125K\ N\cdot m}$$

$$M_O = R \times r$$

$$-87.5j - 125K = 350j \times (xi + yj + zk)$$

$$-87.5j - 125K = 350xK - 350zi$$

$$350x = -125 \Longrightarrow x = -0.357\,m$$

$$-350z = -87.5 \quad z = 0.25\,m$$

لغرض الحصول على قيمة x و z نعوض عن R و Mo
كما مبين في المثال

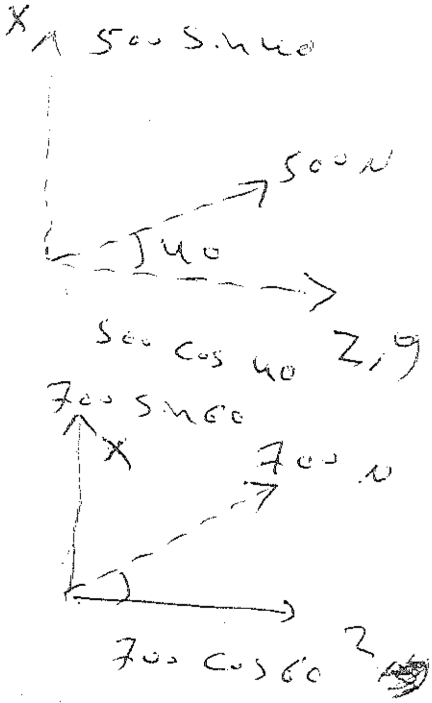
Solution

Ex 2/15 (47)

نلاحظ من الشكل المعطى في المثال، الشكل عبارة عن 3D، لذلك يجب حساب القوى ثلاثة محاور (x, y, z)

$R_x = \sum F_x = 500 \sin 40 + 700 \sin 60$

$= \boxed{928\,N}$



$R_y = \sum F_y$

نلاحظ من الشكل ان القوه 500N في المحور الافقي هي ناتجة من زاوية 40 و 45 اي زاوية مركبة لذلك يجب ان ناخذ بنظر الاعتبار تأثير مقدار 45° لذلك تكون الحل

$R_y = \sum F_y = 600 + 500 \cos 40 \cos 45$

$= 871\,N$

$R_z = \Sigma F_z$

كذلك بالنسبة للمحور z فإن
هناك تأثير زاوية 40 و 45 عند المركبة N 500

$R_z = \Sigma F_z = 700 \cos 60 + 500 \cos 40^\circ \sin 45^\circ$

$= 621\ N$

$\therefore R = 928i + 871j + 621k\ N$

$R = \sqrt{(928)^2 + (871)^2 + (621)^2}$

$= \boxed{1416\ N}$

الآن ممكن حساب العزوم لكل قوة اذا كانت
مائلة لأحد المحاور.

$M = r \times F$

$M_{500} = (0.08i + 0.12j + 0.05k) \times 500$
$(i \sin 40 + j \cos 40 \cos 45 + k \cos 40 \sin 45)$

r is vector from A to B

بعد الضرب نحصل على

$M_{500} = 18.95i - 5.59j - 16.90k\ N.m$

(49)

$$M_{600} = (600 \times 0.06)i + (600 \times 0.04)K$$

$$= 36i + 24K \ N \cdot m$$

$$M_{700} = (700\cos 60 \times 0.03)i - [(700\sin 60 \times 0.06) + (700\cos 60 \times 0.1)]j - (700\sin 60 \times 0.03)K$$

$$= 10.5i - 71.4j - 18.19K \ N \cdot m$$

* من هذا الشكل فيوجد لدينا عزم ملفوظ مقداره 25 N.m لذلك نوجد حساب couple

$$M = 25(-i\sin 40 - j\cos 40\cos 45 - K\cos 40\sin 45°)$$

عكس اتجاه عقرب الساعة

$$= -16.07i - 13.54j - 13.54K \ N \cdot m$$

والآن نجمع العزم لكل القوى

$$M = 49.4i - 90.5j - 24.6K \ N \cdot m$$

$$\therefore M = \sqrt{49.4^2 + 90.5^2 + 24.6^2}$$

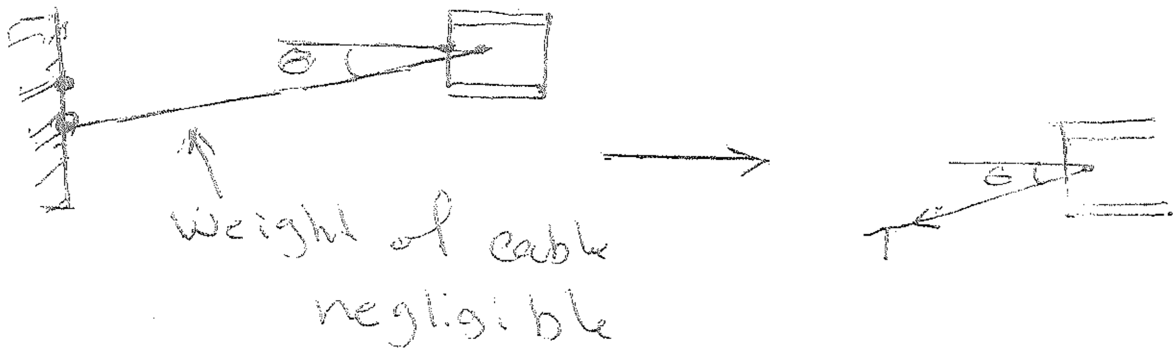
$$= \boxed{106 \ N \cdot m}$$

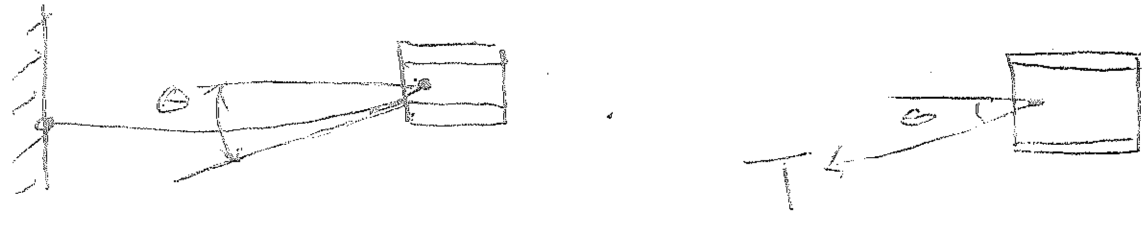
Equilibrium

$R = \Sigma F = 0 \qquad M = \Sigma M = 0$

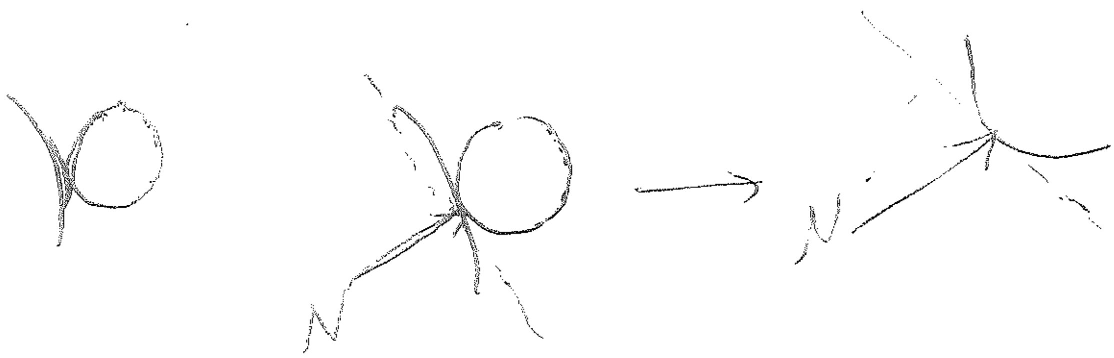
Not: the free body diagram is the most important single step in the solution of problem in mechanics.

① Flexible cable, belt, chain or rope



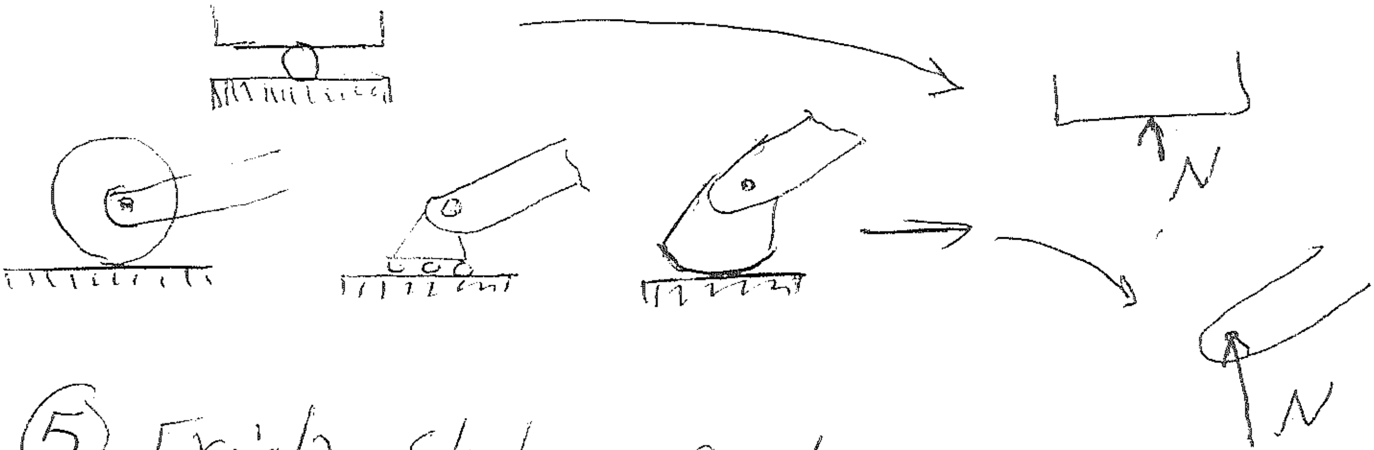


② smooth surface

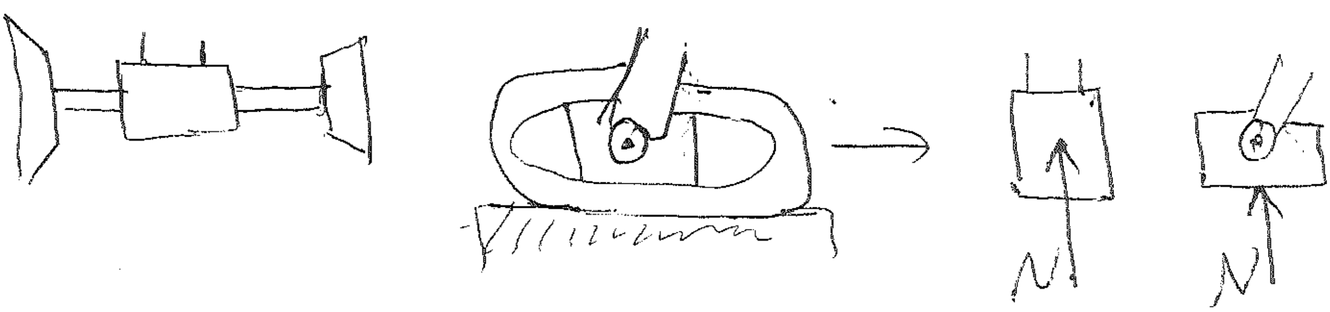


3 Rough surfaces

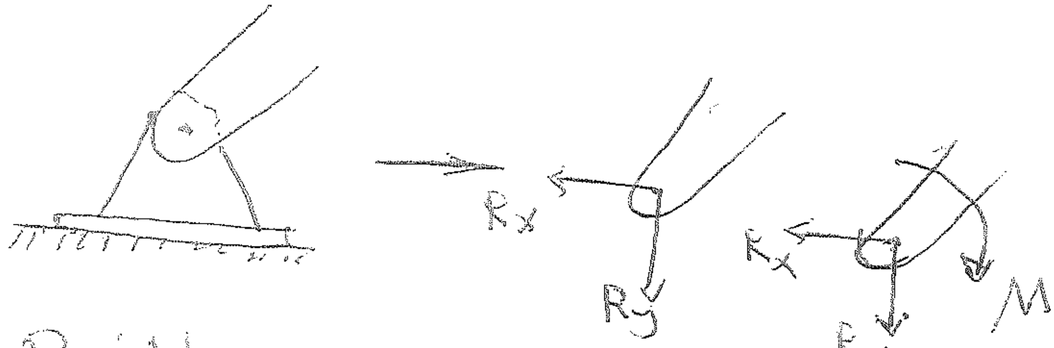
4 Roller support



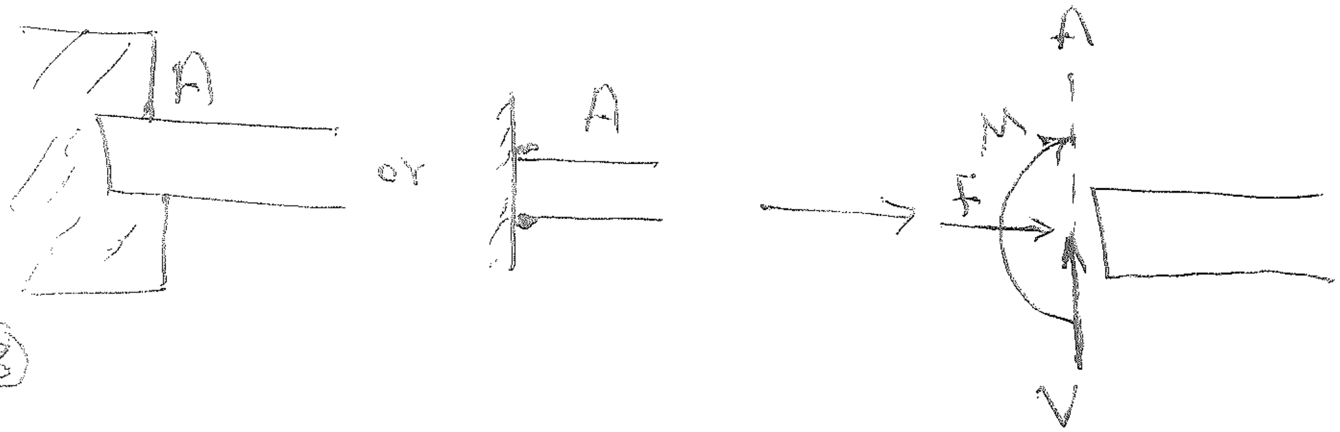
5 Freely Sliding guide



6 Pin Connection

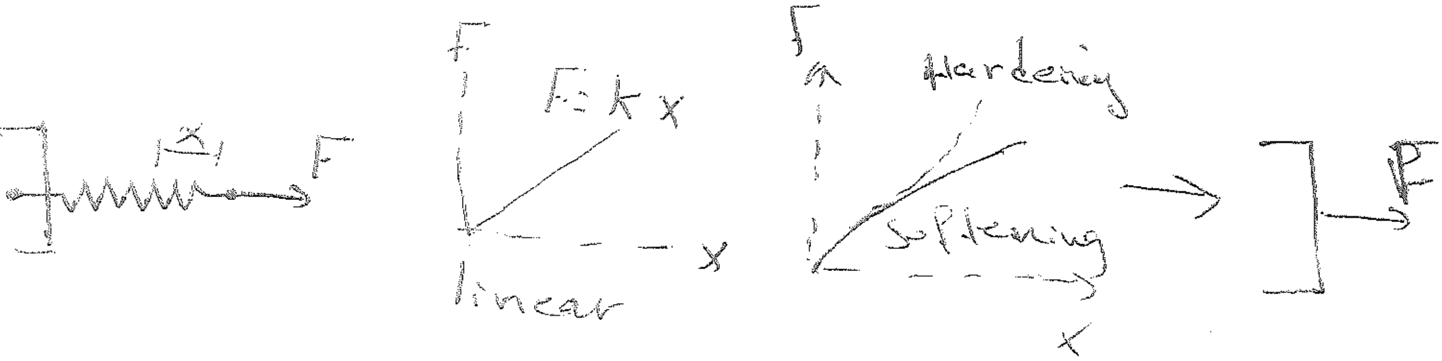


7 Built-in or fixed support



8 Gravitational attraction

9 Spring action



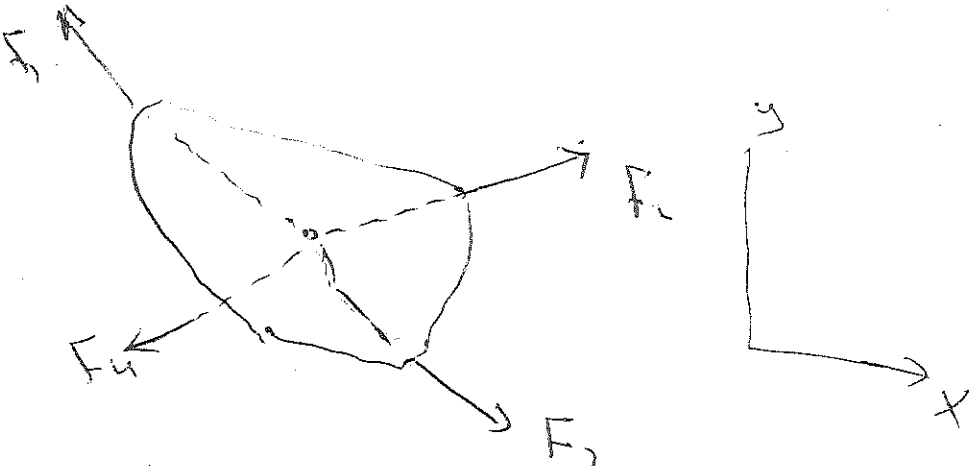
Equilibrium condition

$\Sigma M_o = 0$

$\Sigma F_y = 0$

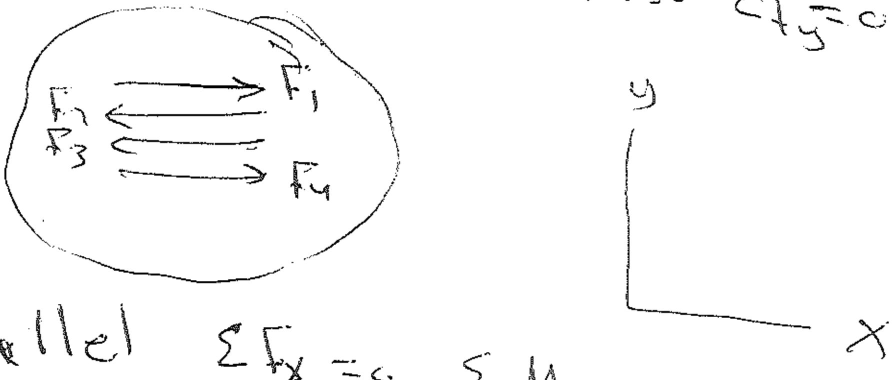
$\Sigma F_x = 0$

(1) Collinear $\Sigma F_x = 0$

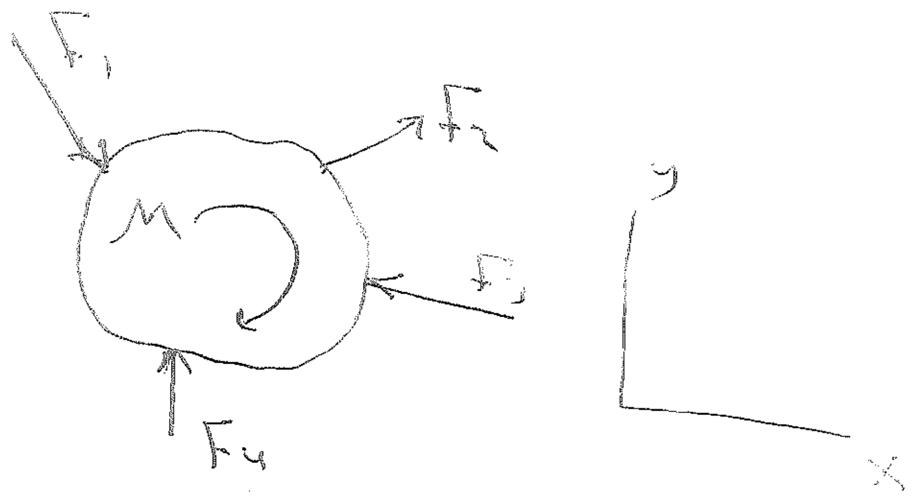


(2) Concurrent at a point $\Sigma F_x = 0$ $\Sigma F_y = 0$

(3)

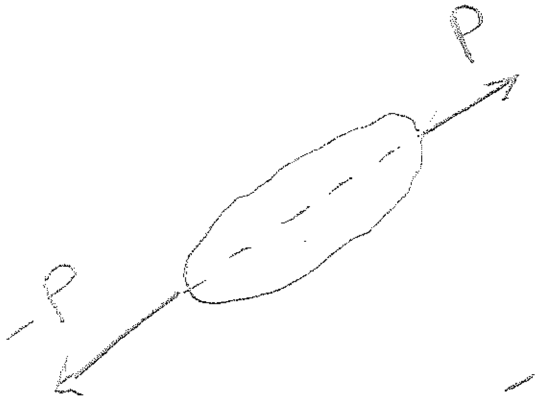


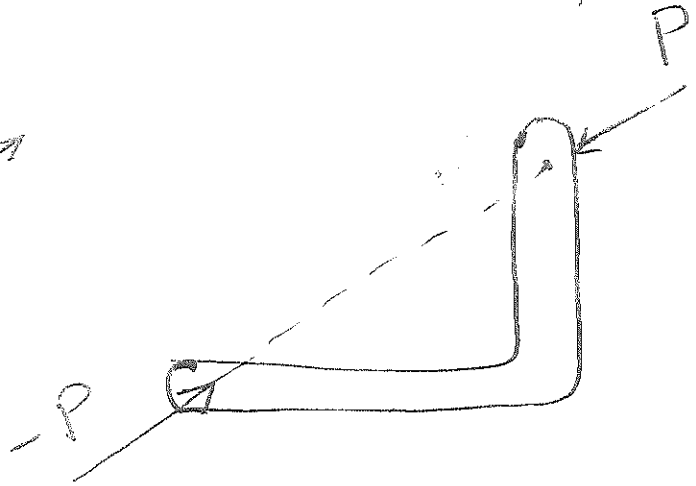
Parallel $\Sigma F_x = 0$ $\Sigma M_z = 0$

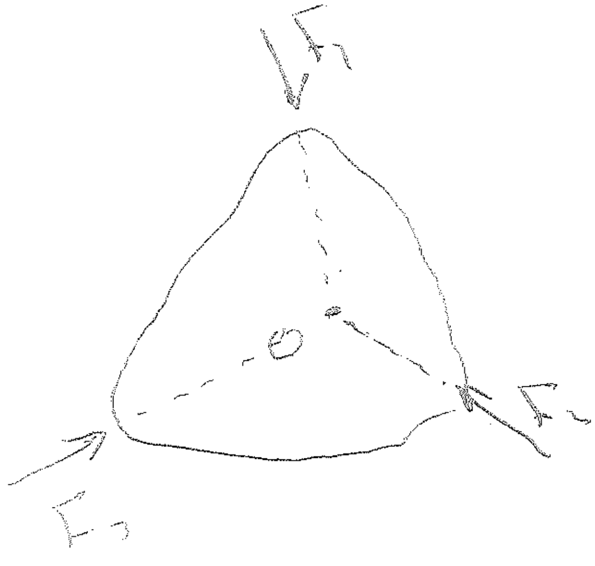


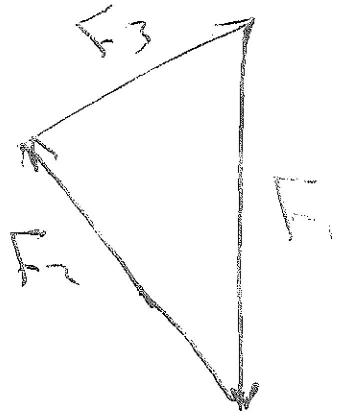
(4) General $\sum F_x = 0$
$\sum F_y = 0$, $\sum M_z = 0$

Two and three Force Members



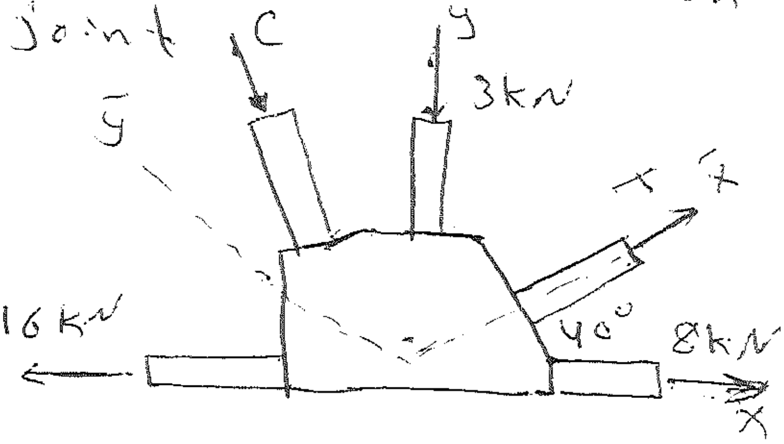






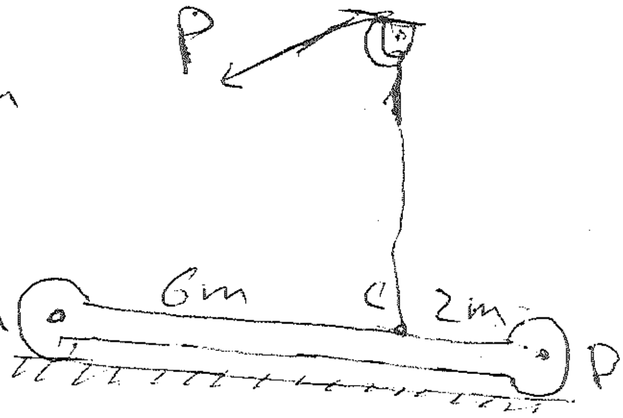
Sample problem 3/1

Determine the magnitudes of the forces C and T which along with the other three forces shown act on the bridge truss joint



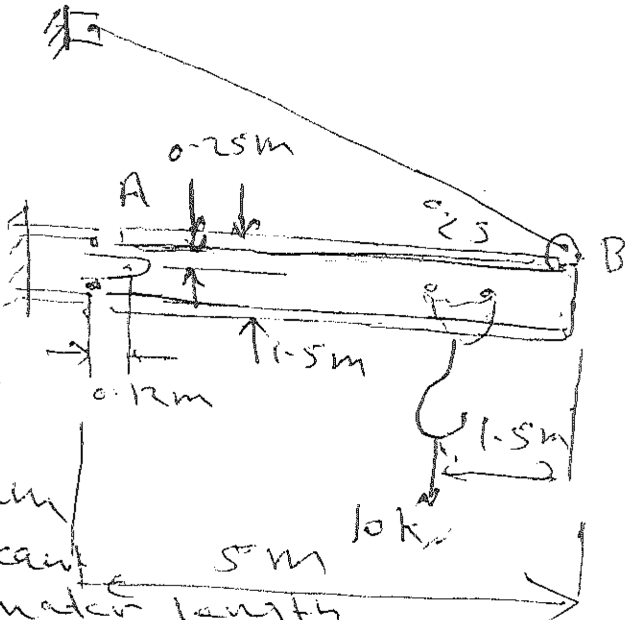
Sample problem 3/3

The uniform 100 kg I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable abc it is desired to elevate end B to a position 3 m above end A. Determine the required tension P, the reaction at A and the angle θ made by the beam with the horizontal in the evaluated position.



Sample problem 3/4

Determine the magnitude of T of the tension in the supporting cable and the magnitude of the force on the pin at A for the jib crane shown. The beam AB is a standard 0.5 m I beam with a mass of 95 kg per meter length

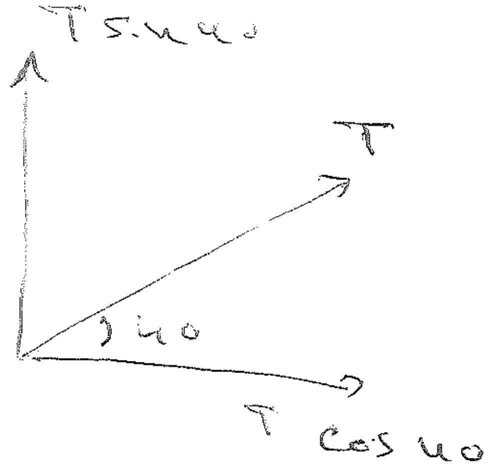


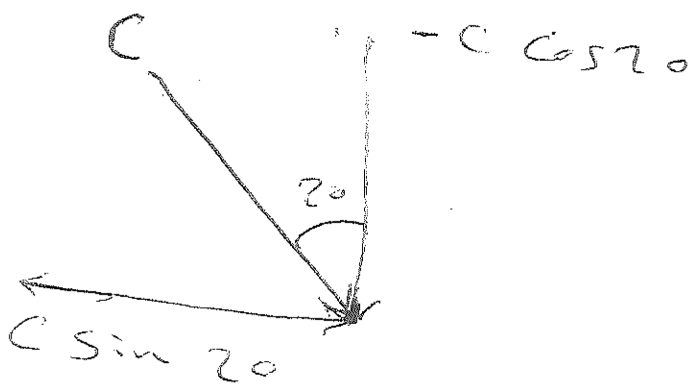
Sample problem 3/1 (Solution)

Method 1

$\Sigma F_x = 0$

$\Sigma F_y = 0$





$\Sigma F_x = 8 - 16 + T\cos 20 + C\sin 20 = 0$

$0.766\,T + 0.342\,C = 8 \longrightarrow (1)$

$\Sigma F_y = 0 \implies T\sin 40 - C\cos 20 - 3 = 0$

$0.643\,T - 0.940\,C = 3 \longrightarrow (2)$

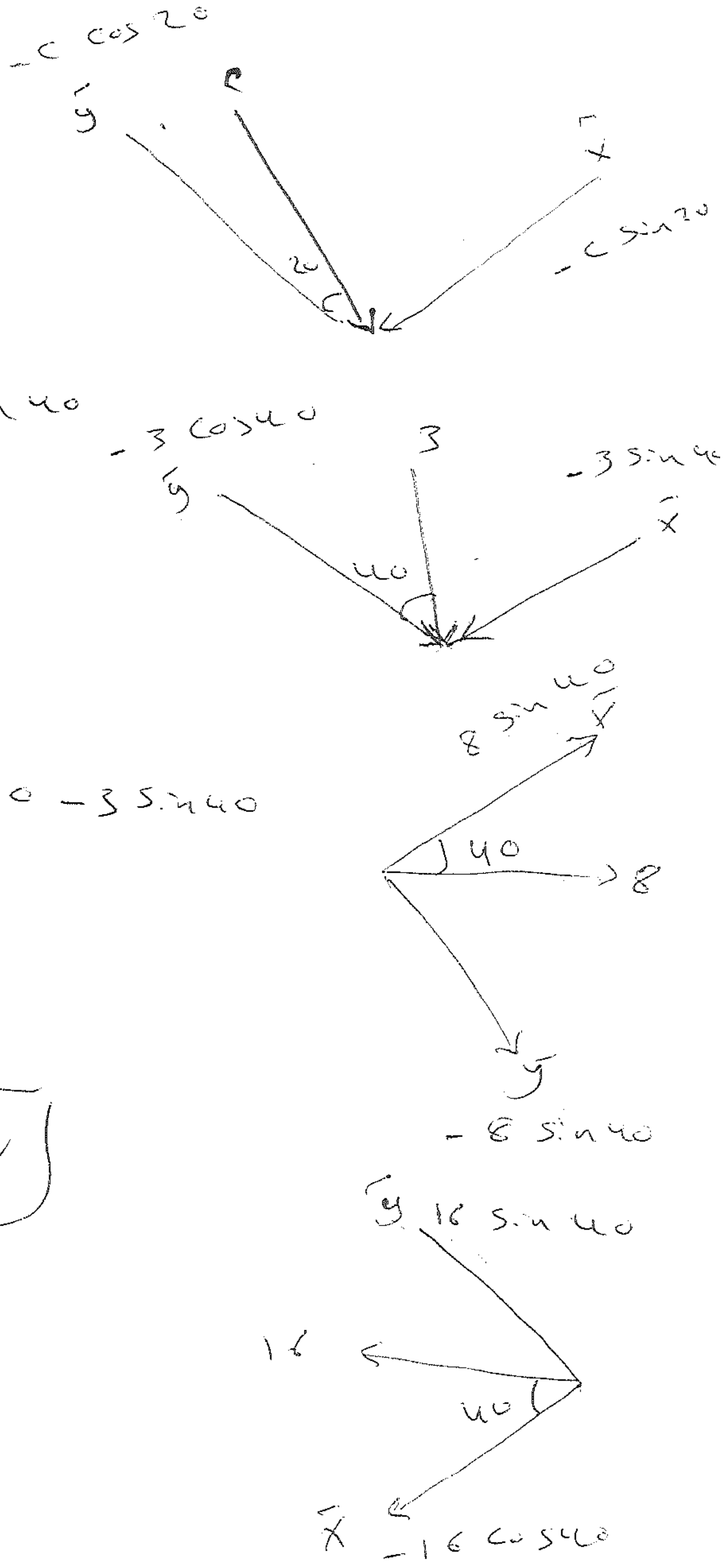
$T = 9.09$ kN

$C = 3.03$ kN

Method 2

$\sum F_{\bar{y}} = 0$

$\sum F_{\bar{x}} = 0$



$\sum F_{\bar{y}} = 0$

$-C\cos 20 - 3\cos 40$

$-8\sin 40 + 16\sin 40$

$\therefore$ $\boxed{C = 3.03\ \text{KN}}$

$\sum F_{\bar{x}} = 0$

$T + 8\cos 40 - 16\cos 40 - 3\sin 40$

$-C\sin 20 = 0$

$\therefore$ $C = 3.03\ \text{kN}$

$\therefore$ $\boxed{T = 9.09\ \text{KN}}$

Method 3

unit vector i and j

$\Sigma F = 0$

$$8i + (T\cos 40)i + (T\sin 40)j - 3j + (C\sin 20)i - (C\cos 20)j - 16i = 0$$

$$8 + T\cos 40 + C\sin 20 - 16 = 0$$

$$T\sin 40 - 3 - C\cos 20 = 0$$

$$T = 9.09\ kN$$
$$C = 3.03\ kN$$

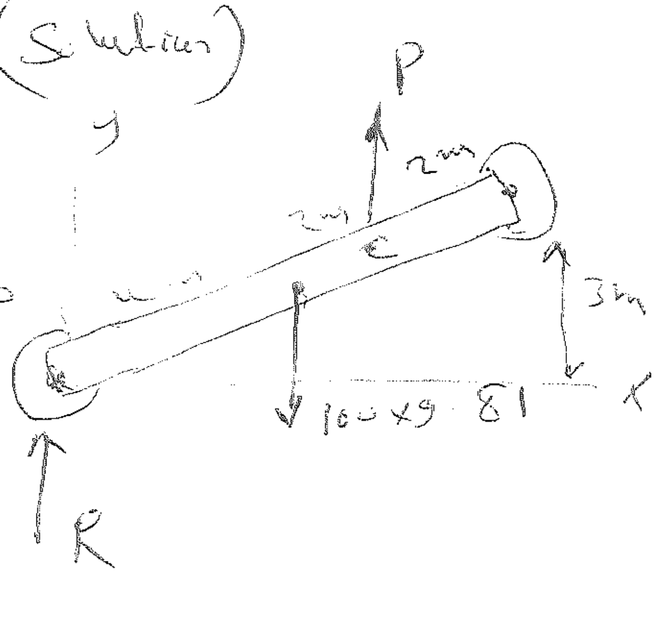
Sample problem 3/3 (Solution)

$\Sigma M_A = 0$

$$P(6\cos\theta) - 981(4\cos\theta) = 0$$

$\theta = 22.0°$

$\therefore P = 654\ N$

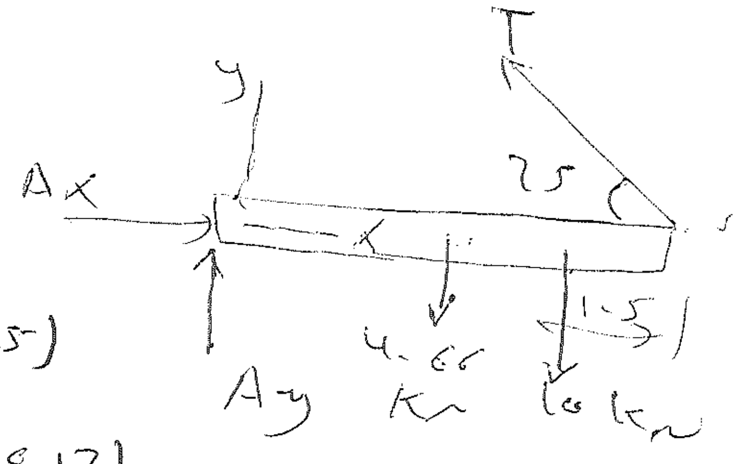


$\Sigma F_y = 0 \quad 654 + R - 981 = 0$

$$R = 327\ N$$

(59)

Sample problem 3/4



Free body diagram

$\Sigma M_A = (T\cos 25)(0.25) + (T\sin 25)(5 - 0.12) = 0$

$T = 19.61\ kN$

$\Sigma F_x = 0$

$A_x - 19.61\cos 25 = 0 \qquad A_x = 17.77\ kN$

$\Sigma F_y = 0$

$A_y + 19.61\sin 25° - 4.66 - 10 = 0$

$A_y = 6.37\ kN$

so $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(17.77)^2 + (6.37)^2}$

$= \boxed{18.88\ kN}$

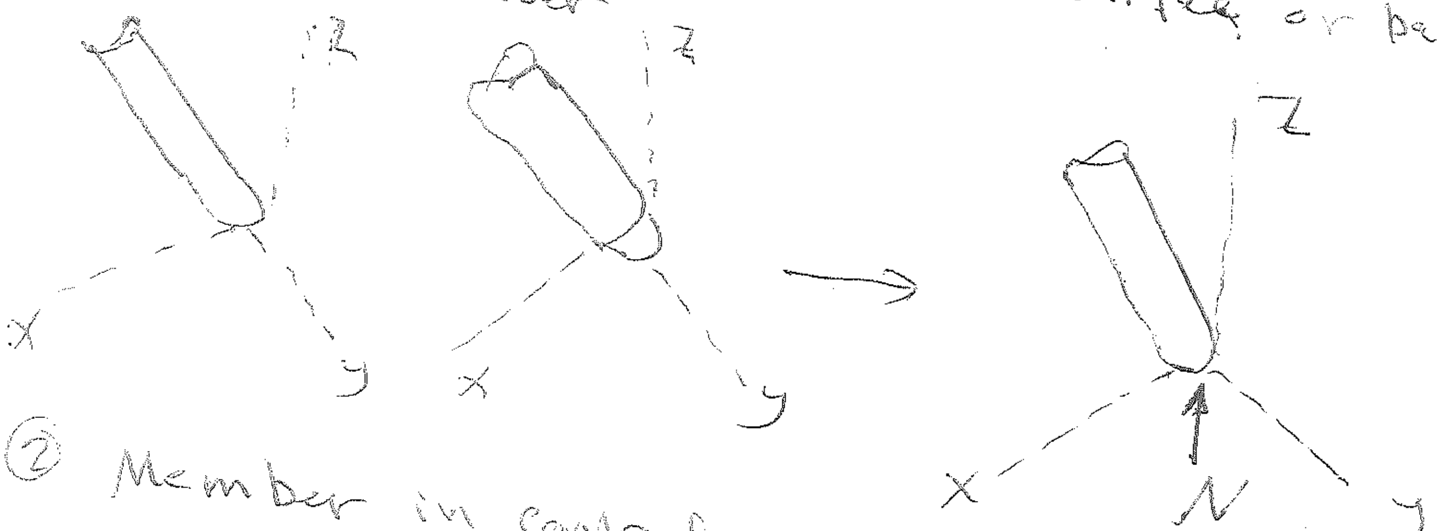
Equilibrium in three dimensions

$$\sum F = 0 \quad \text{or} \quad \left.\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array}\right\}$$

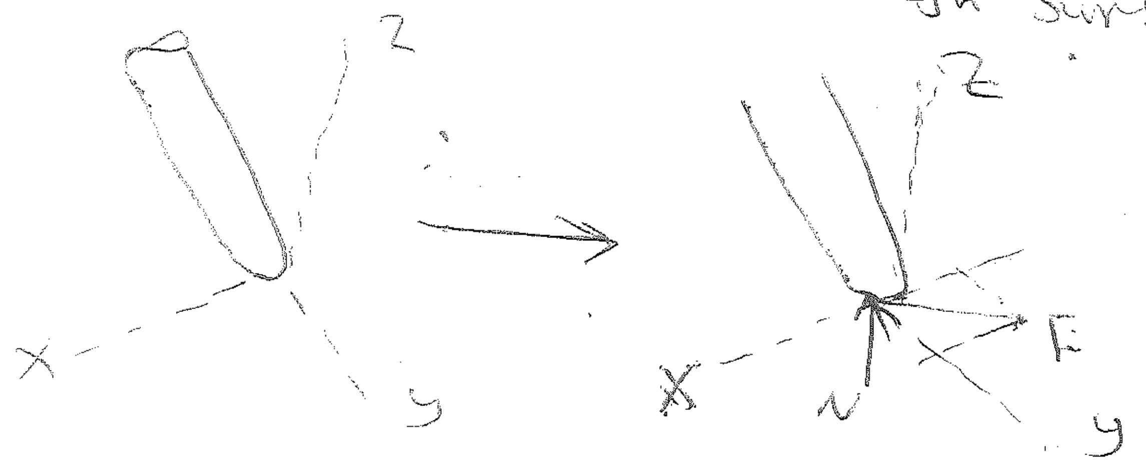
$$\sum M = 0 \quad \text{or} \quad \left.\begin{array}{l} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{array}\right\}$$

Modeling the action of forces in three-Dimensional analysis

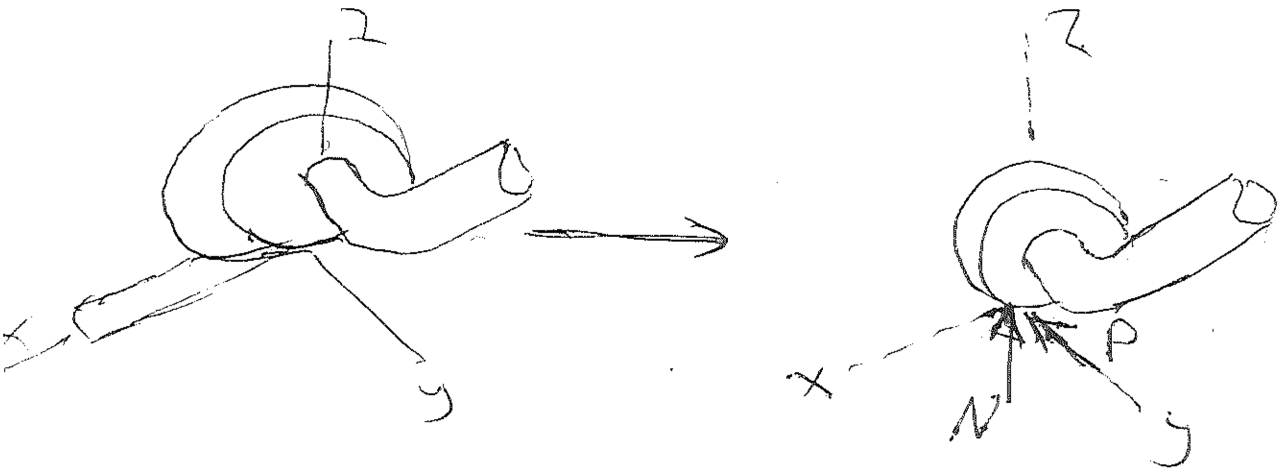
① Member in contact with smooth surface or ball supported member



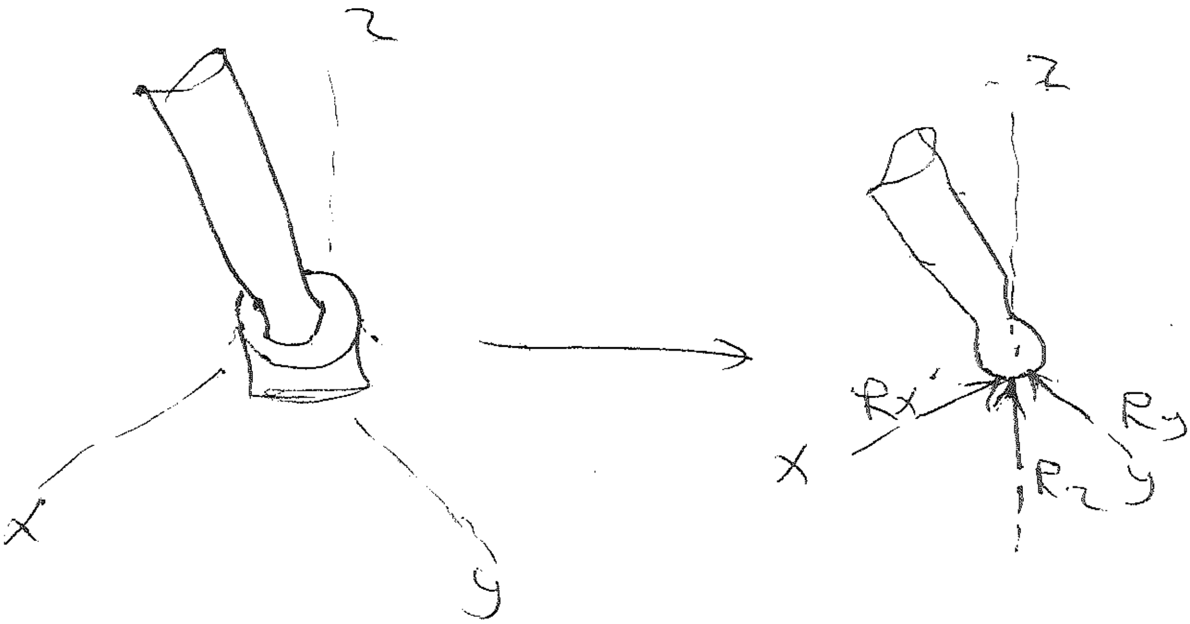
② Member in contact with rough surface



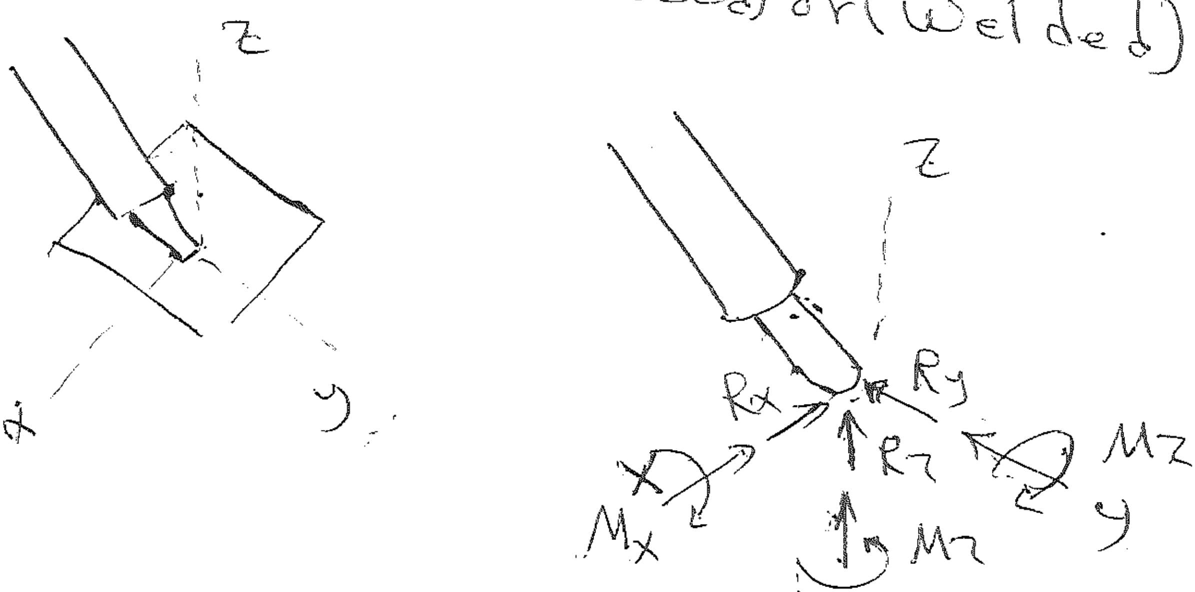
③ Roller or Wheel Support with lateral Constraint



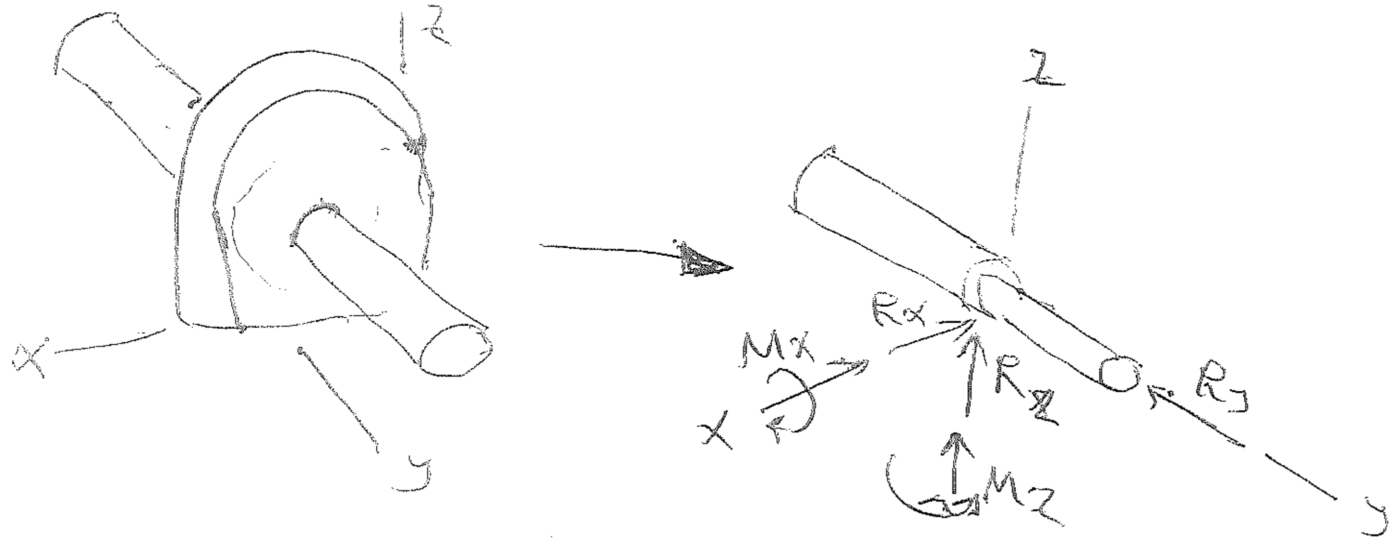
④ Ball-and-Socked joint



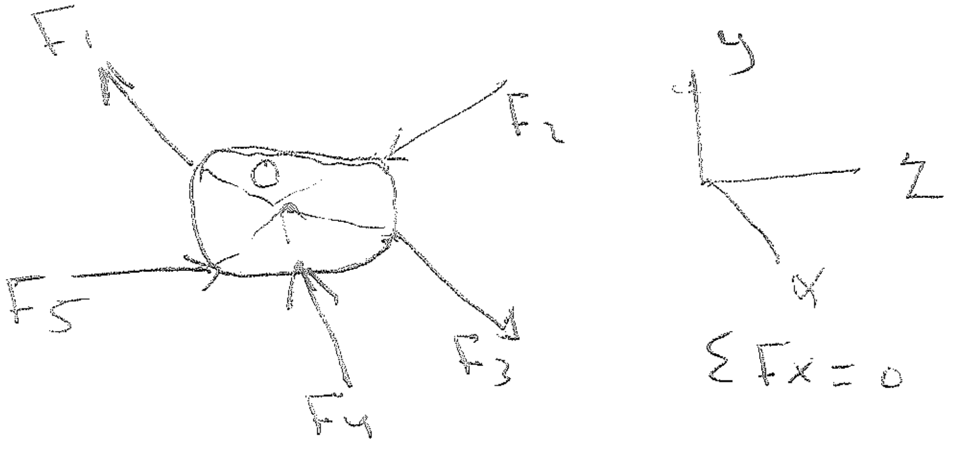
⑤ Fixed Connection (embedded) or (Welded)



6 Thrust - bearing Support

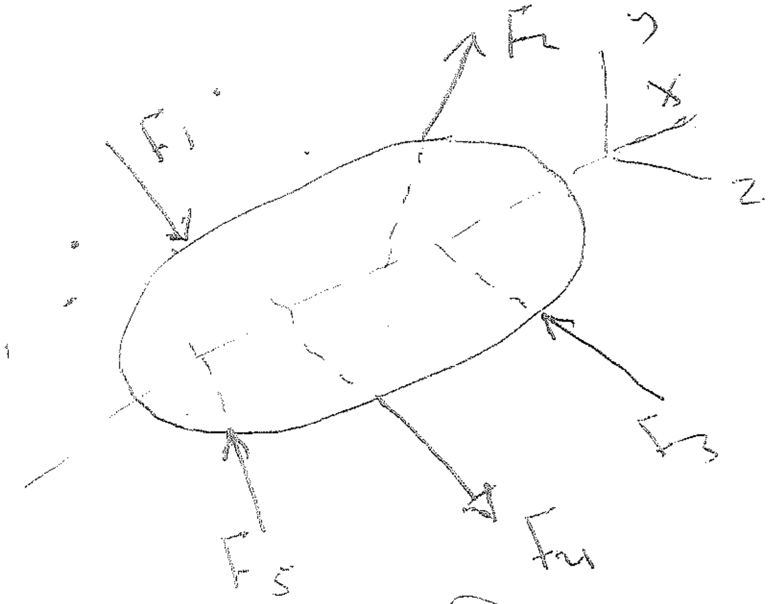


Categaries of equilibrium in 3D



$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

① Concurrent at a point



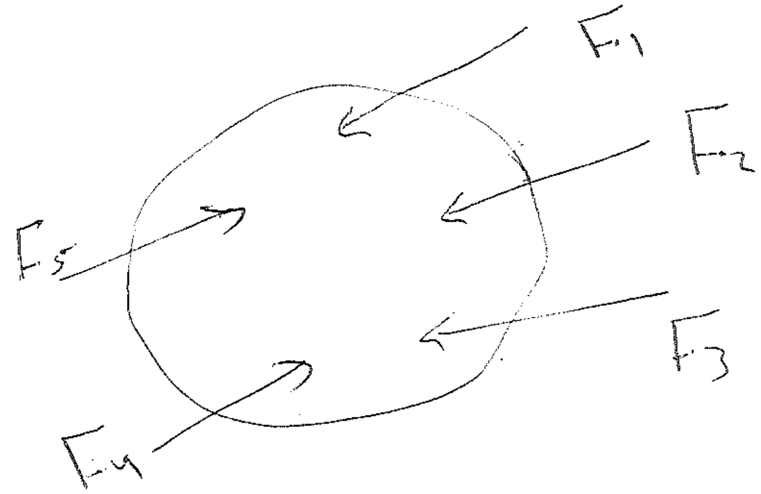
$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$

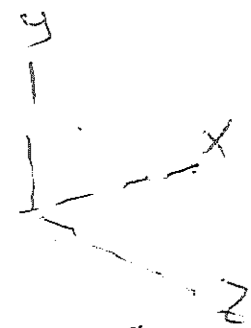
$\Sigma F_y = 0$ $\quad \Sigma M_z = 0$

$\Sigma F_z = 0$

② Concurrent with a line

63

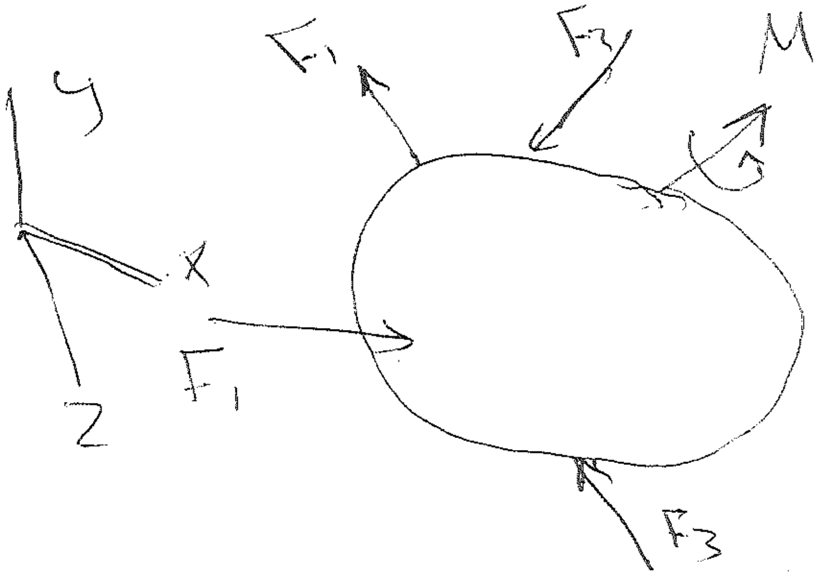




$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$

$\Sigma M_z = 0$

③ Parallel



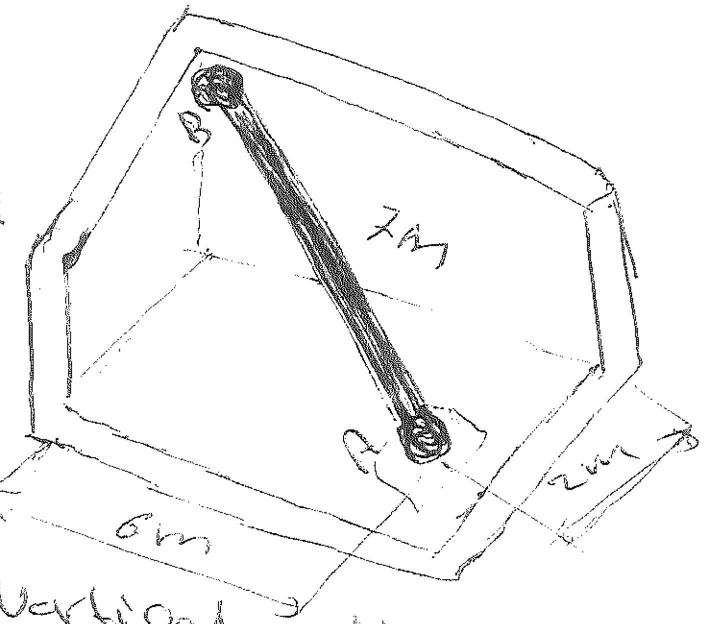
$\Sigma F_x = 0 \quad \Sigma M_x = 0$

$\Sigma F_y = 0 \quad \Sigma M_y = 0$

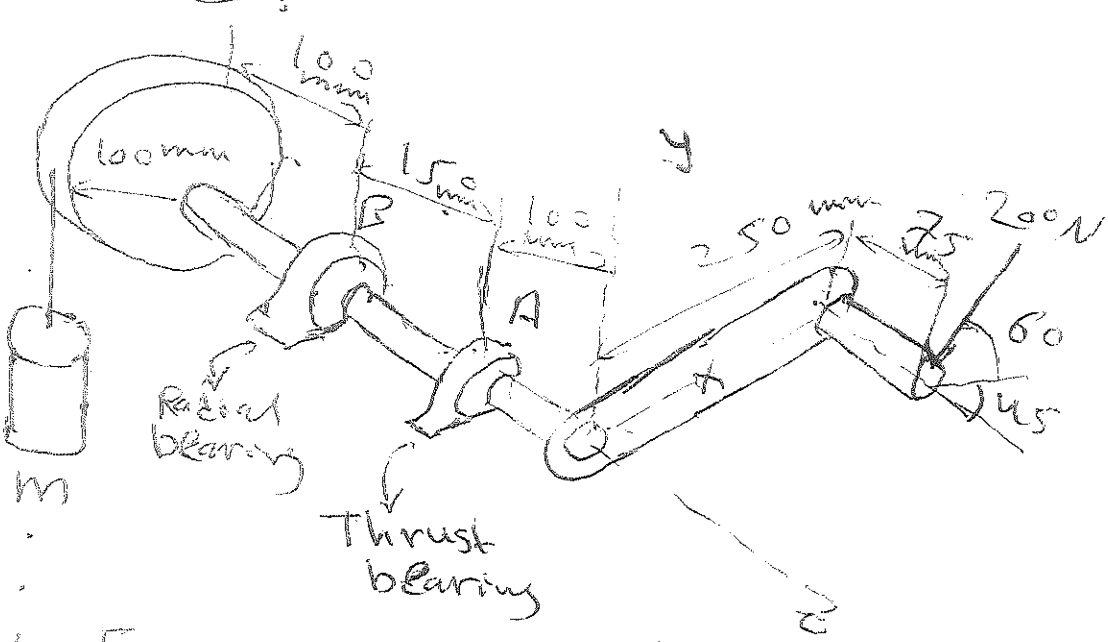
$\Sigma F_z = 0 \quad \Sigma M_z = 0$

Sample problem 3/5

The uniform 7m steel shaft has a mass of 200 kg and is supported by a ball and socket joint at A in the horizontal floor. The ball end B rest against the smooth vertical walls as shown above. Compute the forces exerted by the walls and the floor on the ends of the shaft.



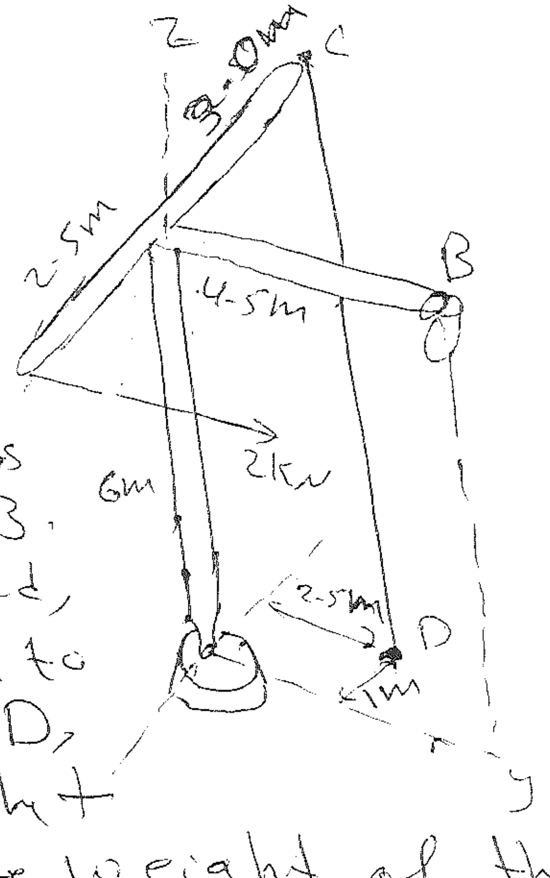
Sample problem 3/6



A 200 N Force is applied to the handle of the hoist in the direction shown in figure above. The bearing

(A) supports the thrust (force in the ~~direct~~ direction of the shaft axis), while bearing B supports only radial load (load normal to shaft axis). Determine the mass m which can be supported and the total radial force exerted on the shaft by each bearing. Assume neither bearing to be capable of supporting a moment about a line normal to the shaft axis.

Sample problem 3/7

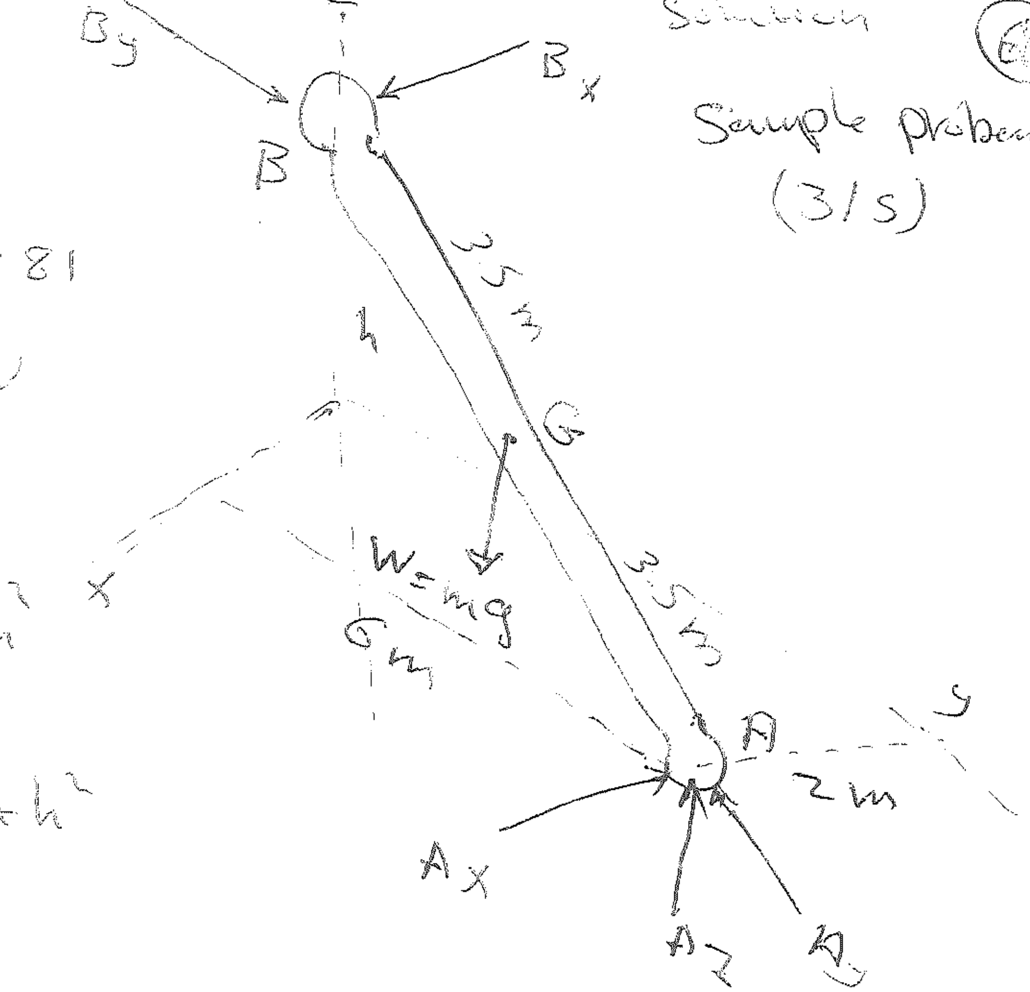


The welded tubular frame is secured to the horizontal x-y plane by a ball and-socket joint at A and receives supports from the loose-fitting ring at B. Under the action of the 2kN load, rotation about a line from A to B is prevented by the cable CD, and the frame is stable in that position shown. Neglect the weight of the frame compared with the applied load and determine the tension T cable, the reaction and the ring and the reaction components at (A)

Vector Solution

Solution (66)

Sample problem (3/5)



$W = mg$

$= 200 \times 9.81$

$= 1962\ N$

$AB^2 = 2^2 + 6^2 + h^2$

$7^2 = 2^2 + 6^2 + h^2$

$\therefore\ h = 3$

$r_{AG} = -1i - 3j + 1.5k\ m$

$r_{AB} = -2i - 6j + 3k\ m$

$\Sigma M_A = 0 \quad r_{AB} \times (B_x + B_y) + r_{AG} \times W = 0$

$(-2i - 6j + 3k) \times (B_x i + B_y j) + (-i - 3j + 1.5k) \times (-1962k) = 0$

$$\begin{vmatrix} i & j & k \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ -1 & -3 & 1.5 \\ 0 & 0 & 1962 \end{vmatrix} = 0$$

(67)

$$\left[(0-3B_y)i+(5890)i\right]-\left[\cancel{(0-3B_x)j+}\ (1962)j\right]+$$

$(0-3B_x)j+$

$$\left[(-2B_y+6B_x\right]=0$$

$$(-3B_y+5890)i+(3B_x-1962)j+(-2B_y+6B_x)=0$$

$$-3B_y+5890=0$$

$$-3B_y=5890 \Longrightarrow \boxed{B_y=1962\ N}$$

$$3B_x-1962=0$$

$$3B_x=1962 \Longrightarrow \boxed{B_x=654\ N}$$

$$\Sigma F=0 \quad (654-A_x)i+(1962-A_y)j+(-1962+A_z)k=0$$

$$\boxed{A_x=654\ N} \quad \boxed{A_y=1962\ N} \quad \boxed{A_z=1962\ N}$$

$$A=\sqrt{A_x^2+A_y^2+A_z^2}=\sqrt{654^2+1962^2+1962^2}$$

$$=\boxed{2850\ N}$$

Scalar Solution (68)

$\sum M_{Ax} = 0 \quad (1962 \times 3) - (3 \times B_y) = 0$

$-3B_y - 5890 = 0 \quad \boxed{B_y = 1962\,N}$

$\sum M_{Ay} = 0 \quad (-1962 \times 1) + (3 \times B_x) = 0$

$3B_x - 1962 = 0 \quad \therefore \quad \boxed{B_x = 654\,N}$

$\sum F_x = 0 \quad -A_x + 654 = 0 \quad A_x = 654\,N$

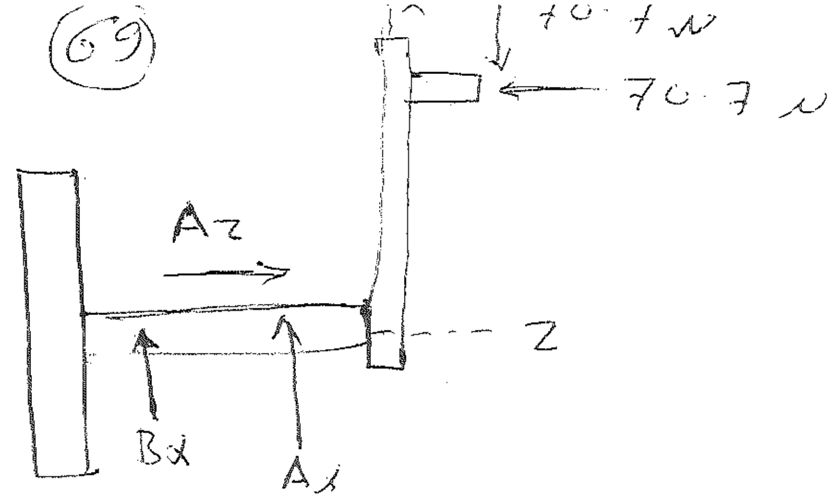
$\sum F_y = 0 \quad -A_y + 1962 = 0 \quad A_y = 1962\,N$

$\sum F_z = 0 \quad A_z - 1962 = 0 \quad A_z = 1962\,N$

$A = \sqrt{654^2 + 1962^2 + 1962^2} = \boxed{2850\,N}$

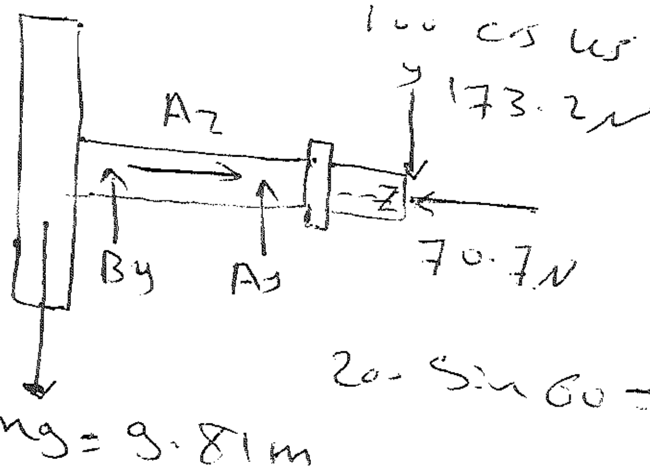
Sample problem 3/6

Solution

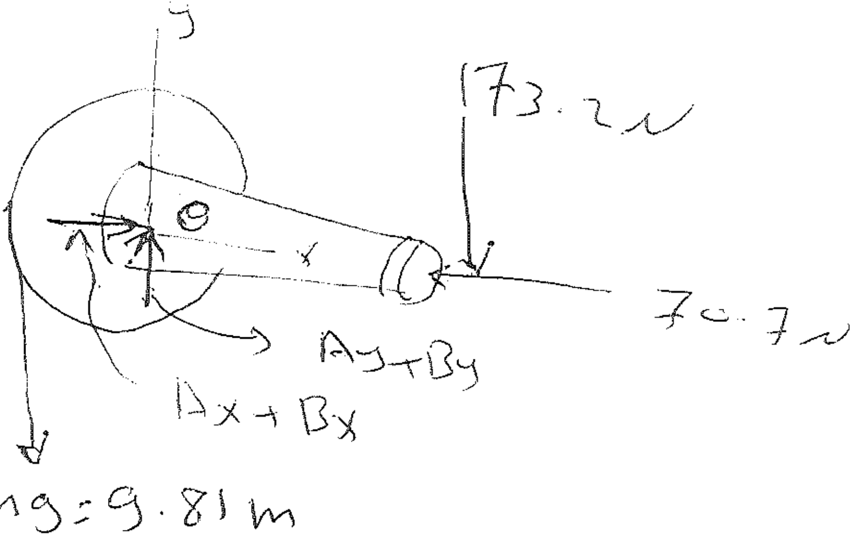


$100 \sin 45 = 70.7\ N$

$100 \cos 45 = 70.7\ N$



$200 \sin 60 = 173.2\ N$



From x-y projection

$\Sigma M_O = 0$

$(100 \times 9.81 m) - (250 \times 173.2) = 0$

$\boxed{m = 44.1\ kg}$

From x – z projection

$\Sigma M_A = 0$: $(150 \times B_x) + (175 \times 70.7) - (250 \times 70.7) = 0$

$\therefore B_x = 35.4\,N$

$\Sigma F_x = 0$ $A_x + 35.4 - 70.7 = 0$ $\therefore A_x = 35.4\,N$

y – z view gives

$\Sigma M_A = 0$ $(150 \times B_y) + (175 \times 173.2) - (250 \times 44.1 \times 9.81) = 0$

$\therefore B_y = 520\,N$

$\Sigma F_y = 0$ $A_y + 520 - 173.2 - (44.1 \times 9.81) = 0$

$\therefore A_y = 88.8\,N$

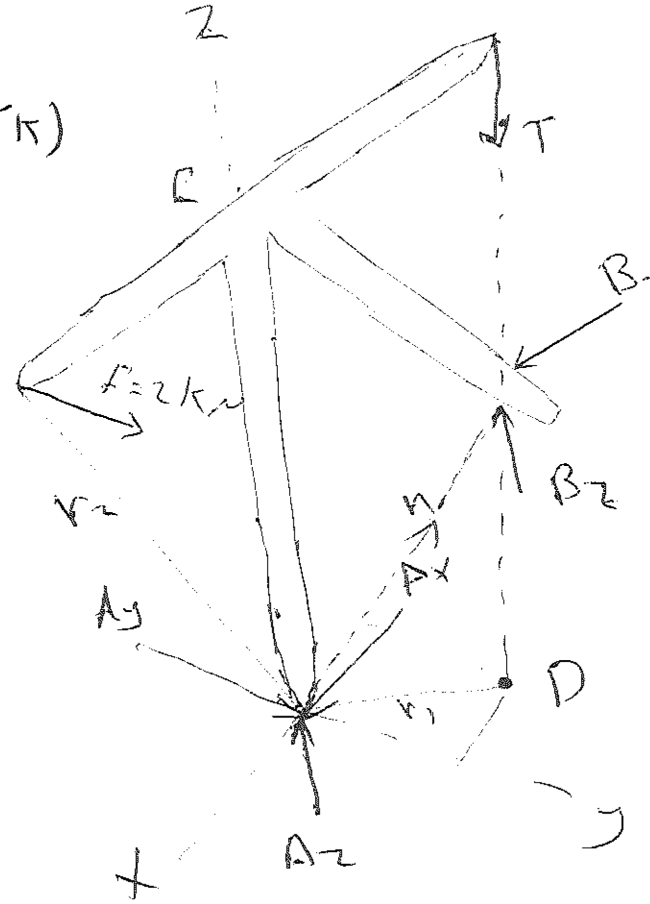
$\Sigma F_z = 0$ $A_z = 70.7\,N$

$A_r = \sqrt{A_x^2 + A_y^2} = \sqrt{35.4^2 + 88.8^2} = \boxed{93.5\,N}$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{35.4^2 + 520^2} = \boxed{521\,N}$

Sample problem 3/7 (71)

Solution



Vector $n = \frac{1}{\sqrt{6^2+4.5^2}}(4.5j+6k)$

$= \frac{1}{5}(3j+4k)$

$r_1 \times T \cdot n$

$r_2 \times F \cdot n$

$\overline{CD} = \sqrt{46.2}$ m

$T = \frac{T}{\sqrt{46.2}}(2i + 2.5j - 6k)$ $\quad F = 2j$

$r_1 = -i + 2.5j$ m $\quad r_2 = 2.5i + 6k$ m

$\Sigma M_{AB} = 0 \quad (-i + 2.5j) \times \frac{T}{\sqrt{46.2}}(2i + 2.5j - 6k) \cdot \frac{1}{5}(3j + 4k) + (2.5i + 6k) \times (2j) \cdot \frac{1}{5}(3j + 4k) = 0$

$-\frac{48T}{\sqrt{46.2}} + 20 = 0$ $\quad \boxed{T = 2.83\ kN}$

$T_x = 0.833$ kN $\quad T_y = 1.042$ kN

$T_z = -2.5$ kN

$\sum M_z = 0$

$2(2.5) - 4.5 B_y - 1.042(3) = 0$ $\boxed{B_x = 0.417\ kN}$

$\sum M_x = 0$ $4.5 B_z - 2(6) - 1.042(6) = 0$

$\boxed{B_z = 4.06\ kN}$

$\sum F_x = 0$

$A_x + 0.417 + 0.833 = 0$ $\boxed{A_x = 1.250\ kN}$

$\sum F_y = 0$

$A_y + 2 + 1.042 = 0$ $\boxed{A_y = -3.04\ kN}$

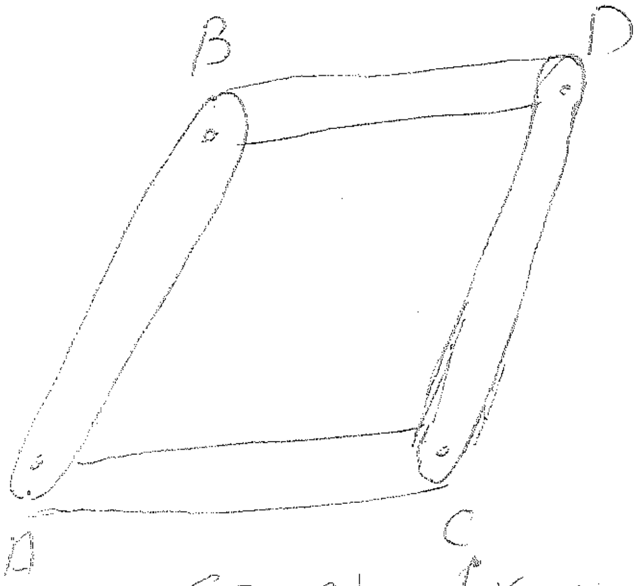
$\sum F_z = 0$

$A_z + 4.06 - 2.50 = 0$ $\boxed{A_z = -1.556\ kN}$

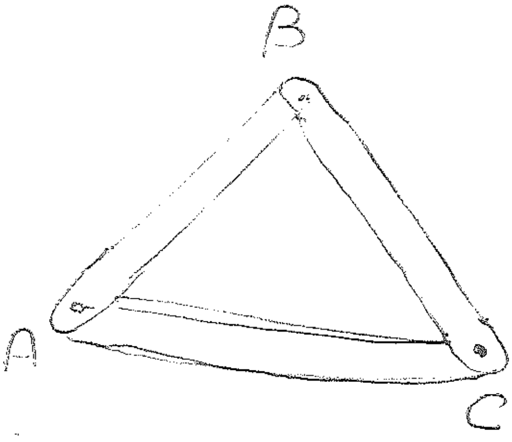
Trusses

الجمالون

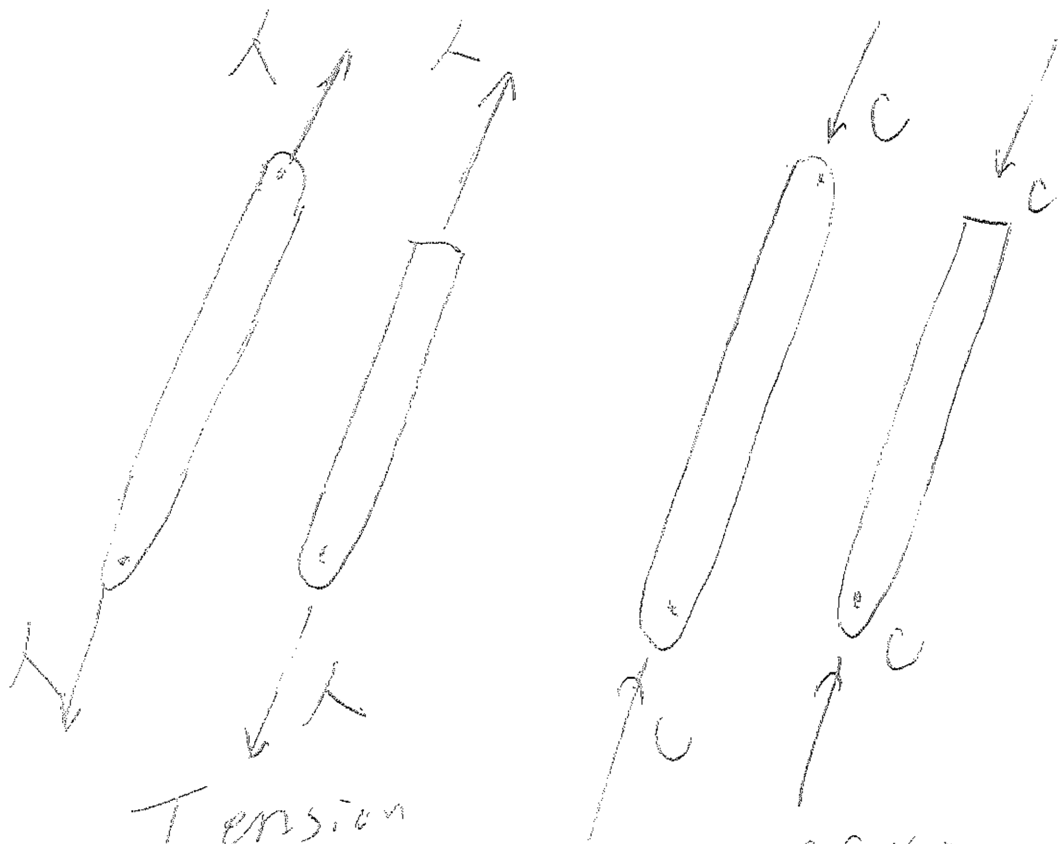
Plane Trusses



Simple truss



Simple truss



Tension

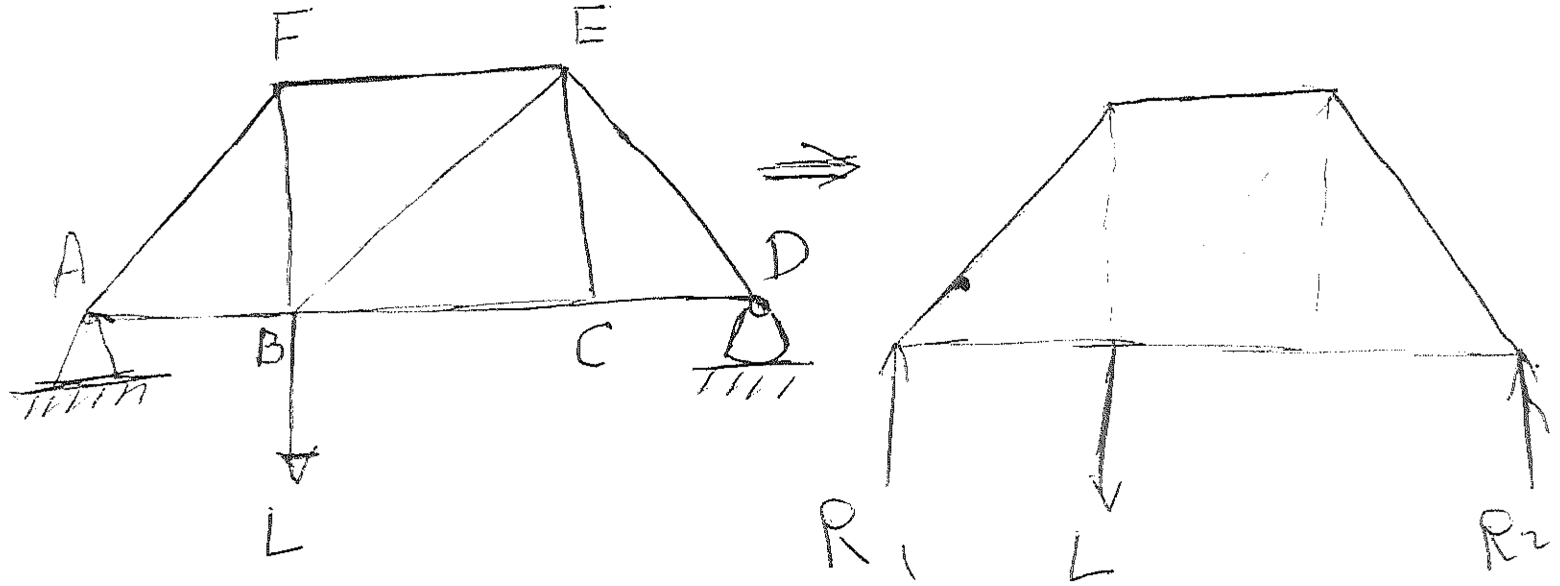
compression

Two forces Members

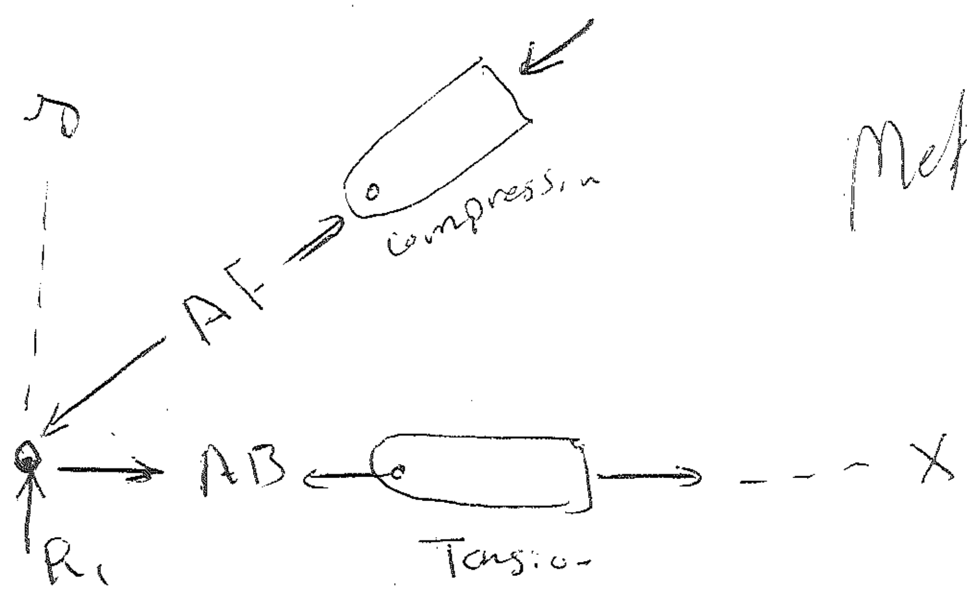
There are two forces of members

1. Tension force.
2. compression force.

Truss connection an supports



For the truss above we should Determine and Calculate R_1 and R_2



Method of joints

P 109

Internal and External Redundancy

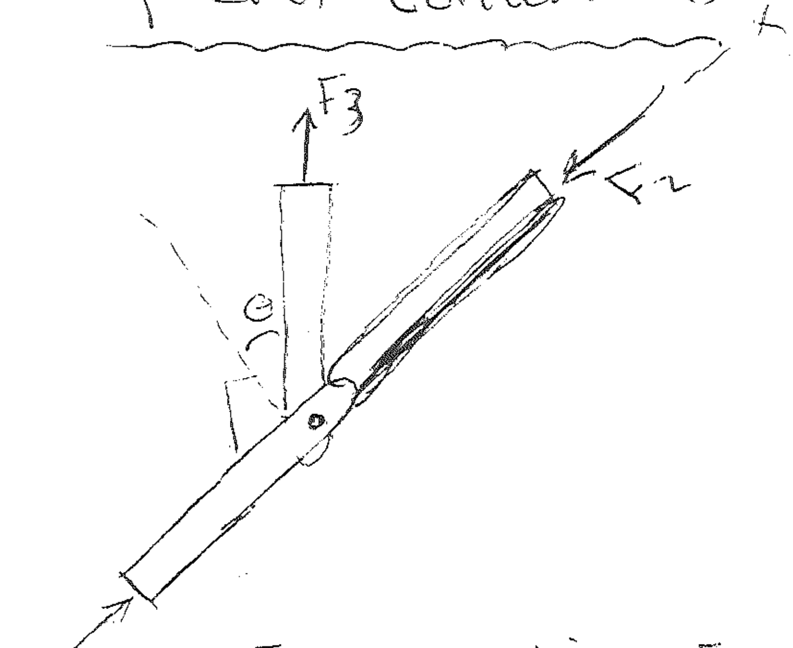
IF plane has more external supports than are necessary to ensure astable equilibrium configuration - we can say that the system has external redundancy.

اذا كان للنظام الدعامات اكثر من تثبيت خارجي اي دعم
غير ضروري لضمان استقرار التوازن يمكن أن نلاحظ بأنه
نظام لديه دعامات خارجية متعددة .

while if a truss has more internal members than are necessary to prevent collapse, then the extra members constitute internal redundancy and the truss is again statically indeterminate

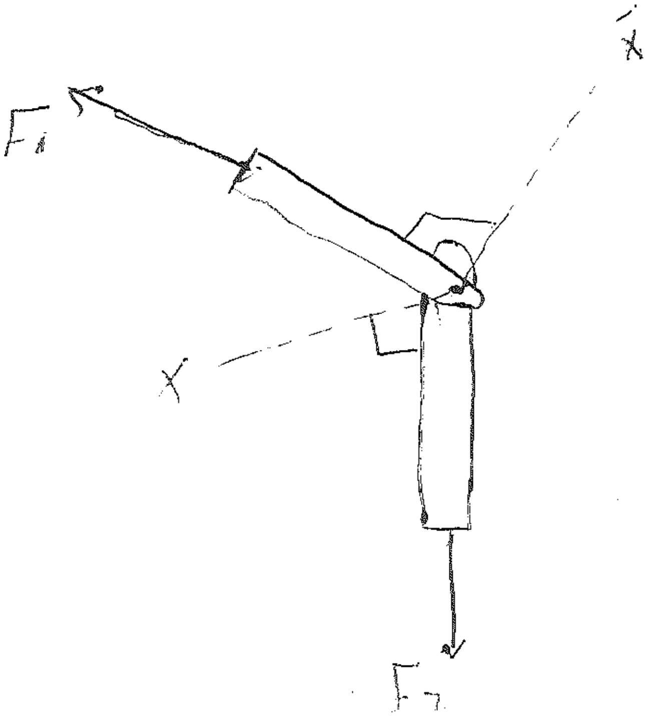
بينما اذا احتوت الدعامة على اكثر من ربط داخلي اكثر
من الضروري لكي يمنع السقوط - هذه الروابط الاضافية
تشكل عملية صلابة داخلية متكررة والدعامة مرة ثانية
تكون معرضة بشكل ثابت ويكون غير محدد بوجود الروابط

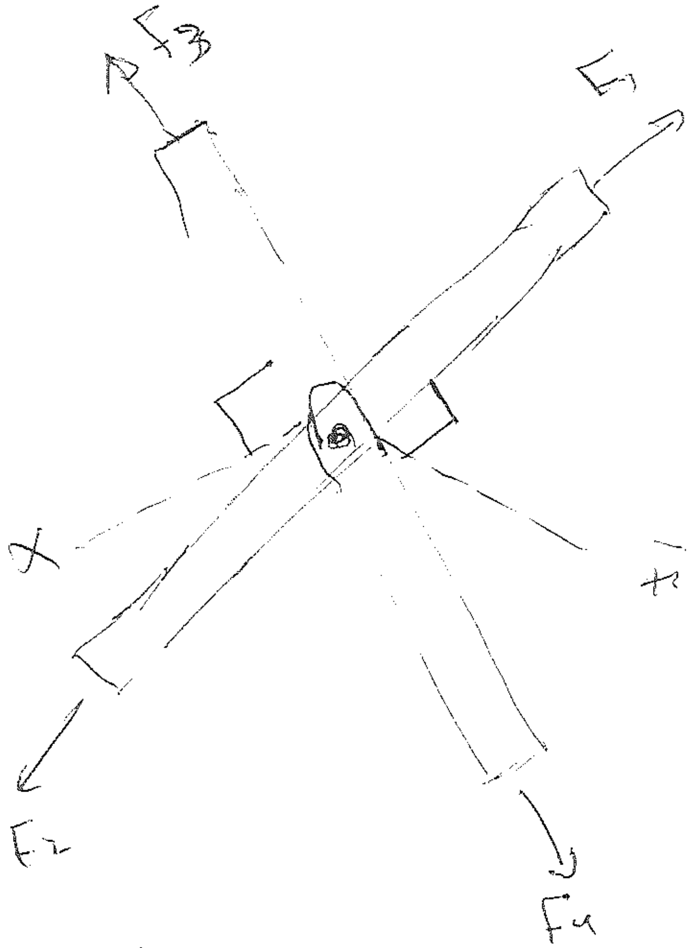
Special conditions



$\Sigma F_y = 0$ requires $F_3 = 0$

$\Sigma F_x = 0$ requires $F_1 = F_2$





$\Sigma F_x = 0$ requires $F_1 = 0$

$\Sigma F_{\bar{x}} = 0$ requires $F_2 = 0$

$\Sigma F_x = 0$ requires $F_1 = F_2$

$\Sigma F_{\bar{x}} = 0$ requires $F_3 = F_4$

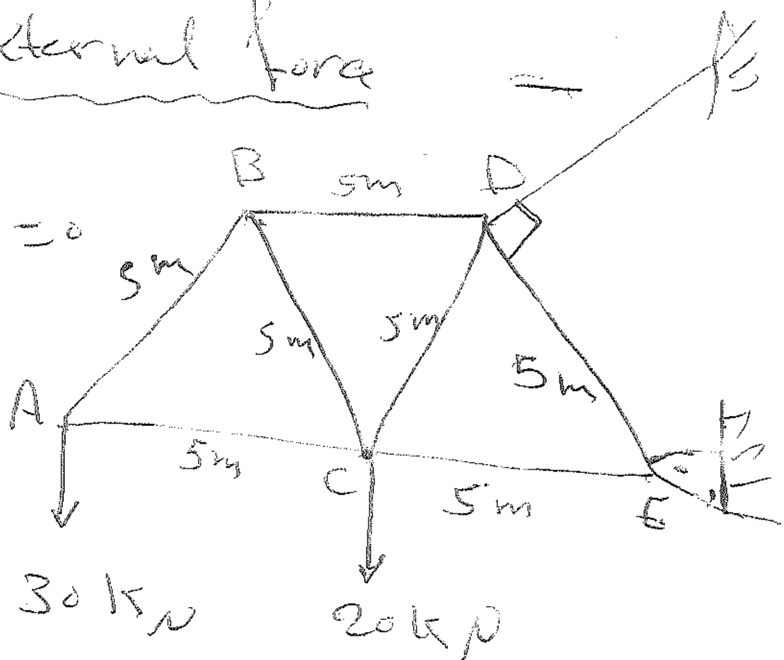
Sample problem (77)

4/1 P (172)

Determine the force in each member

① we should find external force

$\sum M_E = 0$



$5T - (20*5) - (30*10) = 0$

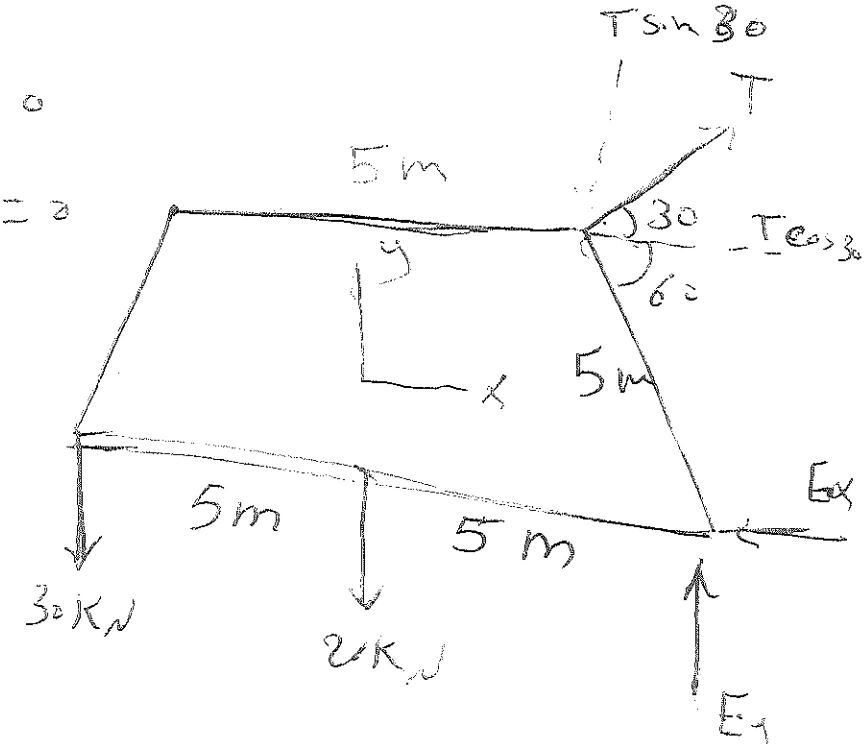
$5T = 100 + 300 = 400$

$T = \frac{400}{5} = \boxed{80\ kN}$

$\sum F_x = 0$

$= T\cos 30 - Ex = 0$

$= 80 \cos 30 - Ex = 0$



$Ex = 80 \cos 30$

$Ex = \boxed{69.3\ kN}$

$\sum F_y = 0$

$T\sin 30 - 30 - 20 + Ey = 0$

$80 \sin 30 - 30 - 20 + Ey = 0$

$Ey = 30 + 20 - 80 \sin 3$

$Ey = \boxed{10\ kN}$

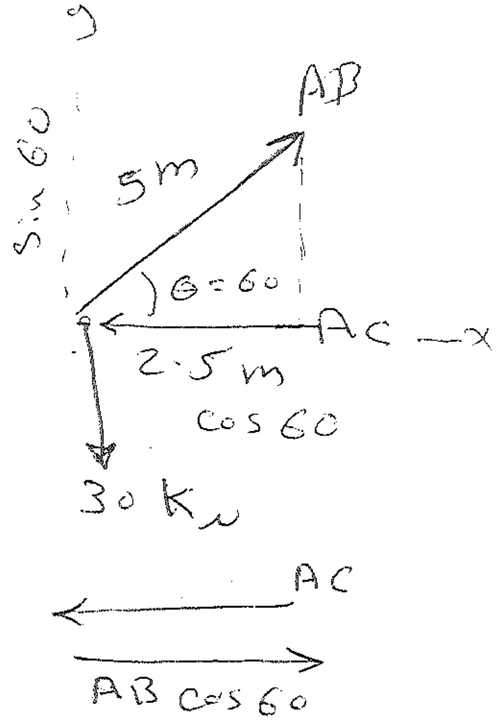
(78)

(2) We can find member forces

$\cos\theta = \frac{2.5}{5} = 0.5$

$\theta = \cos^{-1} 0.5 = 60^\circ$

$\therefore \boxed{\theta = 60^\circ}$



$\Sigma F_y = 0$

$(\sin 60 \times AB) - 30 = 0$

$AB = \frac{30}{\sin 60} = \boxed{34.64\ KN}$ T

$\Sigma F_x = 0$

$AC - (\cos 60 \times AB) = 0$

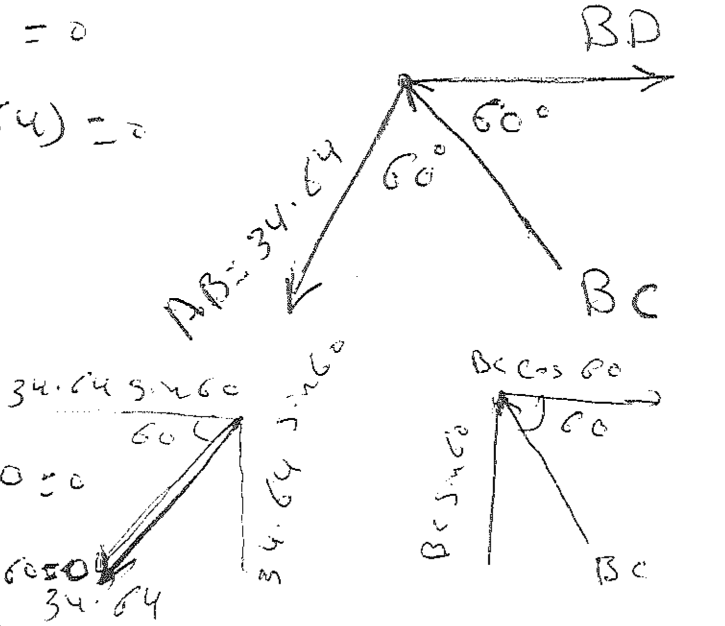
$AC = 34.64 \cos 60 = \boxed{17.32\ KN}$ T

$\Sigma F_y = 0$

$BC \sin 60 - 34.64 \sin 60 = 0$

$0.866 BC - (0.866 * 34.64) = 0$

$BC = \boxed{34.64\ KN}$ C



$\Sigma F_x = 0$

$BD - BC \cos 60 - AB \cos 60 = 0$

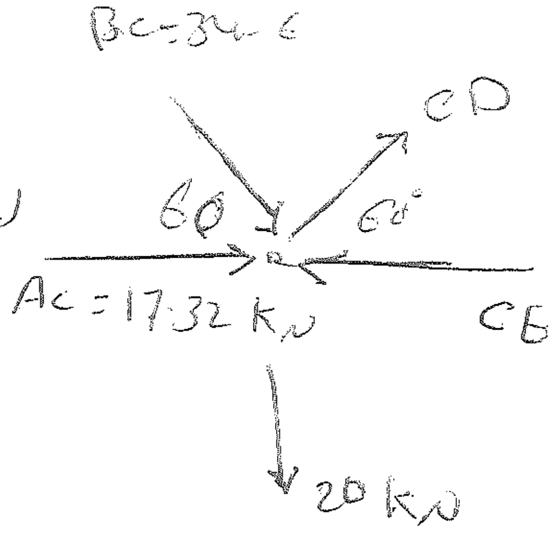
$BD - 34.64 \cos 60 - 34.64 \cos 60 = 0$

$BD = \boxed{34.64\ KN}$ T

$\Sigma F_y = 0$

$\sin 60 \times CD - (\sin 60 \times 34.64) - 20 = 0$

$\boxed{CD = 57.7\,kN}$ T

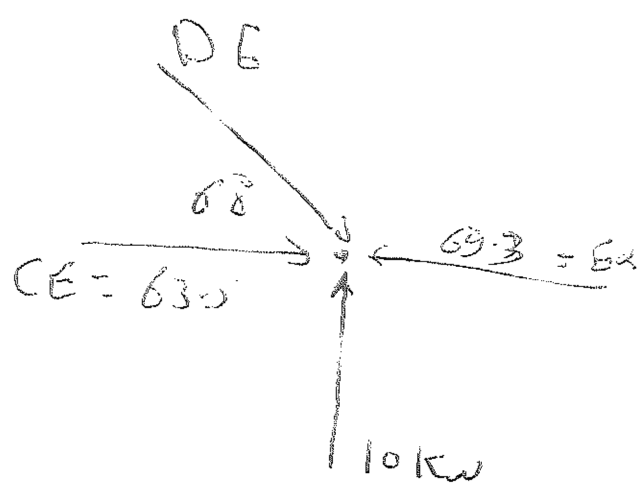


joint C

$\Sigma F_x = 0$

$CE - 17.82 - (\cos 60 \times 34.64) - (\cos 60 \times 57.7) = 0$

$CE = \boxed{63.5\,kN}$ C



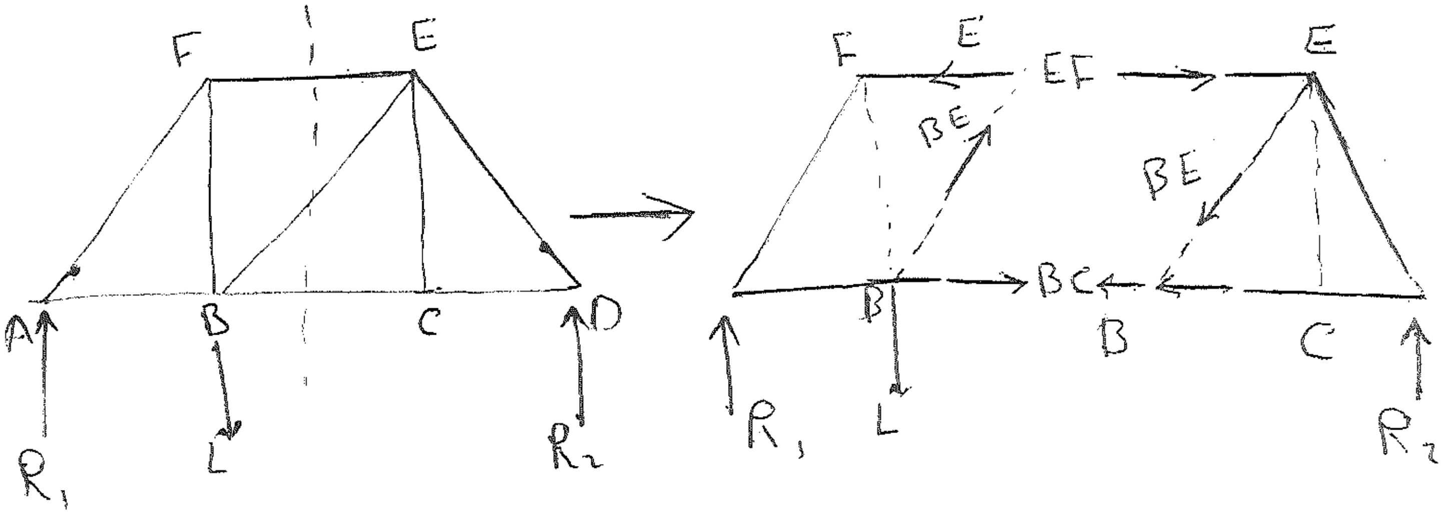
$\Sigma F_y = 0$

$\sin 60 \times DE - 10 = 0$

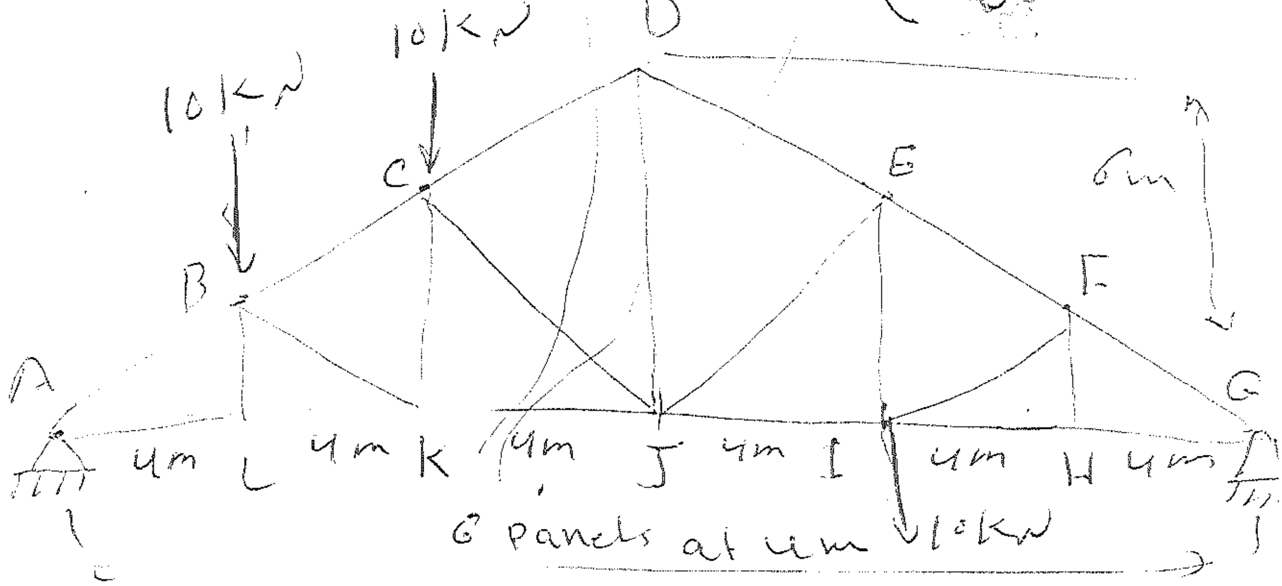
$DE = \boxed{11.55\,kN}$ C

Method of Section

illustration of the method



Sample 4/3 P (182) calculate the force in member (DJ)



Note: Neglect any horizontal components

$$\Sigma M_G = 0 = -(10 \times 8) - (10 \times 16) - (10 \times 20) + (R_A \times 4) = 0$$

$$R_A = \frac{(10 * 8) + (10 * 16) + (10 * 20)}{24}$$

$$\boxed{R_A = 18.333 \text{ KN}}$$

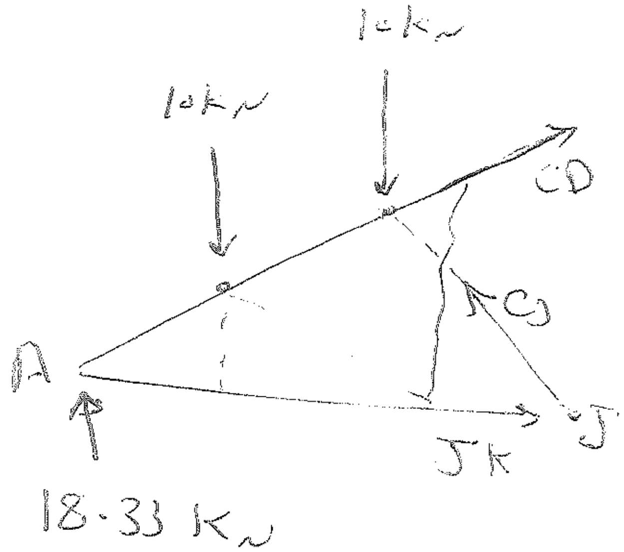
$\Sigma F_y = 0$

$= R_G + 18.33 - 10 - 10 - 10 = 0$

$R_G = 30 - 18.33 = \boxed{11.67 \text{ KN}}$

Section 1

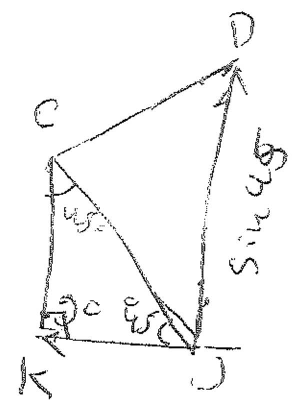
$\Sigma M_A = 0$



$(\sin 45 \times 12)\overset{CJ}{\uparrow} - (10 \times 4) - (10 \times 8) = 0$

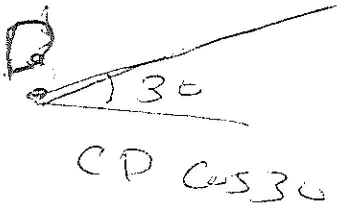
so $CJ = \dfrac{40 + 80}{0.707 \times 12}$

$\boxed{CJ = 14.14 \text{ KN}}$ C



$\Sigma M_J = 0$

$= (CD \cos 30 \times 6) + (18.33 \times 12) - (10 \times 4) - (10 \times 8) = 0$

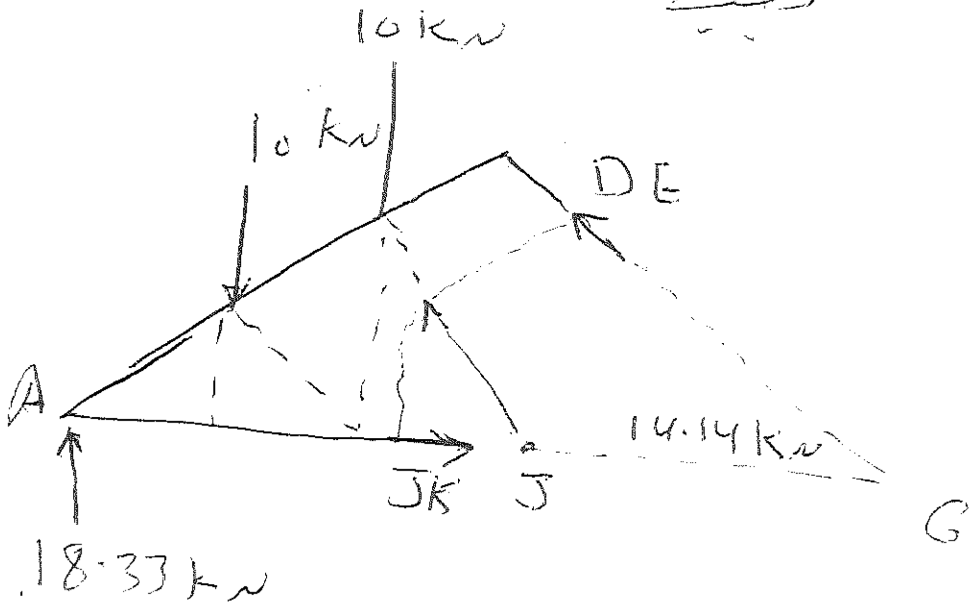


$CD = -18.63 \text{ kN}$

so $CD = \boxed{18.63 \text{ kN}}$ C

لان العزم ل CD حول النقطة J قد اوجدت
باعتبار مركبتين لذلك فان اشارة السالب تعني
أن CD باتجاه الآخر لذلك يجب الاشارة ان
الموجب

Section 2



$\Sigma M_G = 0 = (12 \times DJ) + (10 \times 16) + (10 \times 20) - (14.14 \cos 45 \times 12) = 0$

$\boxed{DJ = 16.67 \text{ kN}}$ T